MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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E. J. STONE, M.A., F.R.S., President, in the Chair.

James Lawson, Houghton House, Ealing;
Emanuel Ristori, 13 Pemberton Crescent, Bayswater; and
Walter George Woollcombe, B.A., The College, Brighton;

were balloted for and duly elected Fellows of the Society.

An Explanation of the Principal Cause of the large Errors at present existing between the Positions of the Moon deduced from Hansen's Tables and from Observation; and the Cause of an Apparent Increase in the Secular Acceleration in the Moon's Mean Motion required by the Tables, or of an Apparent Change in the Time of the Earth's Rotation. By E. J. Stone, M.A., F.R.S.

The errors in the Lunar Theory have been traced to the effects of changes in the *unit of time*, or in the point of reference from which our theoretical longitudes are measured, which have, apparently unconsciously, been introduced from time to time into astronomy with changes in the adopted data.

The argument is clearly seen by a consideration of the different expressions for the longitudes of what may be called the mean Sun which have been adopted for the determination of the sidereal times at mean noon.

If B, H, and V denote the longitudes of the mean Sun, according to Bessel, Hansen, and Le Verrier, we have for 1850 Jan. 1, Paris mean noon $+t$.

$$B = 280^{\circ}\ 46'\ 36''{\cdot}12 + 1296027{\cdot}618184\,.\,t + 0{\cdot}0001221805\,.\,t^2$$

$$H = 280\ 46\ 43{\cdot}20 + 1296027{\cdot}674055\,.\,t + 0{\cdot}0001106850\,.\,t^2$$

$$V = 280\ 46\ 43{\cdot}51 + 1296027{\cdot}678400\,.\,t + 0{\cdot}0001107300\,.\,t^2$$

In all these expressions the unit of time has been *supposed* to be a Julian year of 365·25 mean solar days.

The constant differences, 7″·08 and 7″·39, in B−H and B−V are not unimportant, for they introduce abrupt changes in the record of time; but the differences in the coefficients of t and t^2 show that the *same* unit of time cannot have been adopted in these expressions. The measure of time must be continuous, and the point from which our R.As. are measured, definite. Let, therefore, 1 and $(1+x)$ be the units of time in B and H. Then

$$1296027{\cdot}618184\,.\,t + 0{\cdot}0001221805\,t^2 = 1296027{\cdot}674055\,.\,t\,(1+x) + 0{\cdot}0001106850\,.\,t^2(1+x)^2.$$

If, therefore,

$$n' = 1296027{\cdot}674055$$

$$x = -\frac{0{\cdot}055871}{n'} + \frac{0{\cdot}0001114955\,.\,t}{n'}.$$

To reconcile B and H, therefore, x must contain a variable term. Similar remarks apply to the difference between B and V.

Now let N be the Moon's mean motion referred to 1 as the unit of time, and $(N+\delta N)$ the Moon's mean motion referred to $(1+x)$ as the unit of time. Then

$$(N+\delta N)(1+x) = N,$$

and

$$t\,.\,\delta N = \frac{N}{n'}\{0{\cdot}055871\,.\,t - 0{\cdot}0000114955\,.\,t^2\} = 0''{\cdot}747\,.\,t - 1''{\cdot}54\left(\frac{t}{100}\right)^2.$$

But Hansen determined his mean motion of the Moon so as to force an agreement between his theory and observations reduced with Bessel's unit 1, and his Tables therefore represented the observations well for many years whilst 1 was adopted as the unit of time; but directly the unit of time was changed by the adoption either of H or V, then the effects of the erroneous determination of the Moon's mean motion by Hansen become apparent.

The effect of the change of unit is also shown in the comparison of Le Verrier's Solar Tables with observation, but of course only to about the thirteenth part of the amount shown by the Lunar Tables.

Le Verrier's Tables have been employed in the computation of the Sun's positions in the *Nautical Almanac* since 1864; the Greenwich observations available to me only extend to 1880. The following table exhibits the mean excess in longitude of Le Verrier's tabular places over those deduced from observation. It will be seen that the errors are clearly progressive, and the change per annum appears roughly to be about +0″·052, a result confirming, I think, my opinion of the undesirability of the increase of the mean motion of the Sun by +0″·063.

Year.	Mean Excess in Longitude.	No. of Obs.	Year.	Mean Excess in Longitude.	No. of Obs.
	″			″	
1864	+0·025	122	1873	+0·408	124
1865	+0·136	222	1874	+0·762	106
1866	−0·109	109	1875	+0·728	110
1867	−0·203	93	1876	+0·675	112
1868	+0·245	131	1877	+0·683	87
1869	+0·272	98	1878	+0·816	85
1870	+0·122	126	1879	+0·748	77
1871	+0·306	113	1880	+0·861	115
1872	−0·037	120			

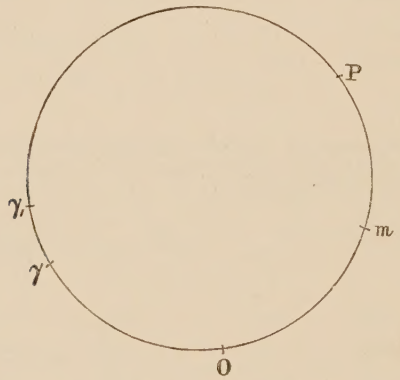
Table showing the agreement of Hansen's Tables with actual observation before *the change in the unit of time, and the progressive error* after *the change in the unit of time introduced in* 1864.

Year.	Mean Value for Year (H−O).	No. of Obs.	Year.	Mean Value for Year (H−O).	No. of Obs.
	″			″	
1847	+1·066	107	1855	−0·871	94
1848	+0·195	113	1856	−0·956	93
1849	−0·465	109	1857	−1·858	115
1850	−0·282	114	1858	−1·975	130
1851	−1·295	109	1860	not compared	
1852	−0·917	109	1861	,,	
1853	−1·629	105	1862	−2·829	85
1854	−1·684	121	1863	1·606	115

Here change in the unit of time took place.

1864	+0·121	111	1873	+8·239	94
1865	+1·271	108	1874	+9·294	117
1866	+2·142	107	1875	+9·867	94
1867	+3·480	89	1876	+9·755	82
1868	+4·117	125	1877	+9·234	105
1869	+4·277	94	1878	+8·219	90
1870	+4·828	102	1879	+9·631	85
1871	+6·955	106	1880	+10·265	101
1872	+7·309	109			

To get rid of the effects of the diurnal rotation I have referred everything to the meridian of some Observatory, as Paris, and discussed the changes of sidereal times at *mean* Paris noons; but as there is apparently some difficulty in following this reasoning, which is based upon the principle of continuity, I give a direct proof of the legitimacy of the method adopted in throwing the changes in the law fixing the position of the point from which our theoretical R.As. are measured, relatively to the Paris meridian at mean noon, on the time in the case of the mean Sun, from whose motion our ideas of mean solar time are derived, and on the determinations of the *mean* motions for the Moon and the planets.



If we assume the time of the Earth's rotation on its axis to be constant—an assumption to be tested by the results—we can always determine the position, relatively to our meridian, of a fixed point on the equator at any definite time. Let this point be O. This is a simplification merely to get rid of the complication of ideas arising from the diurnal motion.

Let m be the mean Sun by whose motion our time is to be measured, and at time t, expressed in any definite unit, let its position be at m, so that

$$\mathrm{O}m = nt$$

(n is a *definite* numerical quantity, since the rotation of the Earth on its axis is supposed to be assigned); and at the same time let P be a planet, and

$$\mathrm{OP} = \mathrm{A} + n't.$$

For practical convenience we measure our angles from γ, where

$$\mathrm{O}\gamma = \mathrm{C} + at + st^2,$$

C, a, and s being assigned constants.

$$\therefore\ \gamma m = C + (a+n)t + st^2$$
$$\gamma P = C + A + (a+n')t + st^2.$$

But suppose, instead of referring our angles to γ, we refer them, in our theoretical investigations, to γ_1, such that

$$O\gamma_1 = C + \delta C + (a+\delta a)t + (s+\delta s)t^2;$$

then

$$(1)\quad \gamma_1 m = C + \delta C + (a+n+\delta a)t + (s+\delta s)t^2,$$

t having the *same* value as before.

$$(2)\quad \gamma_1 P = C + \delta C + A + (a+\delta a+n')t + (s+\delta s)t^2.$$

Now, if we know of the changes δC, δa, δs, and allow for them, we shall correctly determine the relative motion of P measured from γ_1; but if we do not know of these changes, and do not allow for them, how shall we force an agreement between observation and theory? Clearly we shall have to throw the error on the time measured from (1), and on the mean motions of the planets in (2), and we shall thus partially force an agreement, but not a complete one, between observations referred to γ and theory referred to γ_1.

First, let t be the true time deduced either from γm or $\gamma_1 m$; and let $(t+\delta t)$ be the observed time deduced from (1) when δC, δa, and δs are neglected, or when $\gamma_1 m$ is forced to agree with observations referred to γ; and let B be the epoch, x the mean motion, and yt^2 the empirical secular correction required to make $\gamma_1 P$, deduced with the time $(t+\delta t)$, agree with γP; we then have

$$\gamma_1 P = B + x(t+\delta t) + (s+\delta s+y)t^2;$$
$$= C + A + (a+n')t + st^2.$$

But from (1) we have

$$O = \delta C + (a+n)\delta t + \delta a \,.\, t + \delta s \,.\, t^2$$
$$\delta t = \frac{-(\delta C + \delta a \,.\, t + \delta s \,.\, t^2)}{a+n},$$

and

$$\gamma_1 P = B - \frac{x \,.\, \delta C}{a+n} + x\left(1 - \frac{\delta a}{a+n}\right)t + \left(s + y + \delta s - \frac{\delta s \,.\, x}{a+n}\right)t^2$$
$$= C + A + (a+n') \,.\, t + st^2$$
$$\therefore\ B - \frac{x \,.\, \delta C}{a+n} = C + A$$
$$x\left(1 - \frac{\delta a}{a+n}\right) = (a+n')$$

$$s+y+\delta s-\frac{\delta s\,.\,x}{a+n}=s$$

$$\therefore\ x=(a+n')\left(1+\frac{\delta a}{a+n}\right)$$

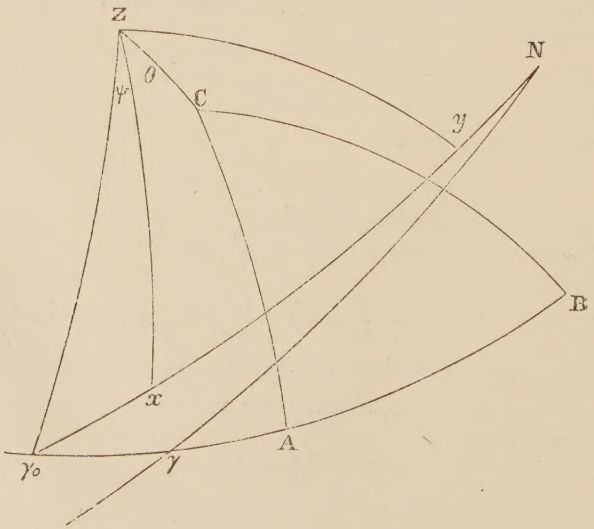
$$B=C+A+\delta C\left(\frac{a+n'}{a+n}\right)$$

$$y+\delta s=\delta s\left(\frac{a+n}{a+n}\right)$$

The correction required to γ P deduced directly from the erroneous time to make it agree with the observations referred to γ is therefore

$$-(\delta C+\delta a\,.\,t+\delta s\,.\,t^2)\left(\frac{a+n'}{a+n}\right),$$

which is the condition upon which the investigation in my paper is based as deduced directly from the consideration of the differences in the units of time necessary to secure continuity in the adopted expressions for sidereal time at mean noon. The possibility of such errors arises from the indirect way in which our theories are compared with observation. Numerical values of the constants which fix the relative changes in the planes and points of reference and the mean motion of Sun have to be adopted and carried into the theories; but the observed R.As. are deduced from the times of transit by clocks whose errors are determined by the differences between the times of transit by the clock and R.As. measured from the point of intersection of the ecliptic and equator, which point must, for accurate work, be the same as that adopted in the theoretical expressions. I have shown, however, by the discordant values of B, H and V, that no definite point, much less the true point, has been practically adopted in our theories.



The breaks of continuity in our measures which have necessarily taken place with the adoption of different units through the introduction of small changes in the Sun's mean notion, and in the secular terms which express the relative motions of the planes of reference, are clearly shown as follows :—

Let x, y, Z be the projections of three rectangular axes fixed in space, xy being the plane of the ecliptic and x the point of intersection of the ecliptic and equator at the instant from which our time is measured ; and at the time t let γ_0AB be the plane of the equator, γN the plane of the ecliptic, C the projection of the polar axis of the Earth, and CA the meridian of the Paris Observatory.

Let

$$\text{ZC} = \theta ; \qquad \gamma_0 \text{Z}x = \psi ; \qquad \gamma_0 \text{A} = \phi ;$$

n = angular velocity of the Earth on its axis.

Then

$$n = \frac{d\phi}{dt} - \frac{d\psi}{dt} . \cos \theta .$$

This equation is exact.

And

$$\phi = \text{C} + nt + \int \frac{d\psi}{dt} . \cos \theta . dt,$$

where C is some constant to be determined from observation. From the theory of the motion of the Earth about its centre of gravity, we have, to the second power of t,

$$(1) \quad \psi = at + bt^2 + \Psi,$$
$$(2) \quad \theta = \omega_0 + ft^2 + \Omega ;$$

where a, b, ω_0, f are constants, Ψ and Ω periodic functions.

$$\therefore \quad \int \frac{d\psi}{dt} . \cos \theta . dt = at \cos \omega_0 + bt^2 \cos \omega_0 + \Psi . \cos \omega_0,$$

and

$$\phi = \text{C} + (n + a \cos \omega_0) . t + bt^2 \cos \omega_0 + \Psi . \cos \omega_0.$$

But from the relative motion of the Sun around the Earth, we have

$$(3) \quad x\text{N} = \alpha - \beta t ; \qquad (4) \quad \gamma_0\text{N}\gamma = \gamma t + \epsilon t^2,$$

where α, β, γ, and ϵ are constants.

From (1), (2), (3), and (4) we obtain

$\gamma_0\gamma = ut + u't^2$; where u and u' are constants.

But

$$\gamma A = \phi - \gamma_c \gamma = s, \text{ suppose};$$

$$s = C + (n + a \cos \omega_o - u)t + (b \cos \omega_o - u')t^2 + \Psi \cos \omega_o.$$

This equation fixes the law of variation of γ the point from which the R.As. are measured, relatively to A, a fixed point on the Earth's surface; and nothing but confusion can result from any alteration in this expression unless the changes in the constants by which it is produced are consistently carried through all our work.

Now suppose the mean longitude of the Sun

$$= L_o + Nt + \lambda t^2.$$

Let

$$n + a \cos \omega_o - u - N = 365{\cdot}25 \times 2\pi;$$

this fixes the unit of time where

n = angular velocity of Earth on its axis,

a = luni-solar precession,

ω_o = obliquity at 1850, Jan. 1,

u = intercept on Equator between ecliptic-planes,

n' = Sun's sidereal mean motion in longitude,

p = general precession.

$N = n' + p.$

All these quantities, when expressed numerically, have to be referred to a Julian year of 365·25 mean solar days. But since the rotation of the Earth on its axis, and the Sun's mean motion in longitude are physical facts, we must have

$$\frac{n}{n'} = C,$$

where C is a constant to be determined from observation; and this relationship must hold good *whatever unit of time be adopted*, and be carried into the equation of condition which fixes the unit of time from the relative motions of the Earth on its axis, and the Sun's mean motion in R.A.

Similarly we may write

$$a \cos \omega_o - u - p = An';$$

where A is some constant.

Our equation of condition which defines the unit of time then becomes

$$n'(C - 1 + A) = 365{\cdot}25 \times 2\pi. \qquad (2)$$

This equation shows at once that we cannot change n' without changing our unit of time. We can of course write

$$(n' + \delta n')(C - 1 + A) = 365{\cdot}25 \times 2\pi, \qquad (3)$$

and still *call* the new unit of time a *Julian year*, but in this case the length of a Julian year in (3) is not the same as in (2); and if t_1 and t_2 are the lengths of the two so-called Julian years, we must have

$$t_1 n' = t_2 (n + \delta n'); \tag{4}$$

therefore

$$t_1 = t_2 \left(1 + \frac{\delta n'}{n'}\right). \tag{5}$$

From which it directly follows that if a period be expressed by t Julian years under system (2), it will exceed the period expressed by t Julian years under system (3) by

$$\frac{\delta n'}{n'} . t. \tag{6}$$

I now proceed to show how it has been frequently assumed that the error due to a change $\delta n'$ is only

$$= \frac{\delta n'}{n} . t, \text{ instead of } \frac{\delta n'}{n'} . t.$$

Let n and $n + \delta n$ be the angular velocities of the Earth on its axis, corresponding to the two values n' and $(n' + \delta n')$ of the Sun's mean motion. Then

$$n = 365{\cdot}25 \times 2\pi + n' + p + u - a \cos \omega_0; \tag{7}$$

but it is then usually assumed that

$$n + \delta n = 365{\cdot}25 \times 2\pi + n' + \delta n' + p + u - a \cos \omega_0, \tag{8}$$

from which of course would follow

$$\delta n = \delta n',$$

and

$$\frac{\delta n}{n} = \frac{\delta n'}{n};$$

but upon this I must remark that since t_1 and t_2 cannot *both* be put $= 1$, we must have instead of (8), the equation of condition

$$n + \delta n = 365{\cdot}25 \times 2\pi (1 + x) + n' + \delta n' + (1 + x)(p + u - a \cos \omega_0). \tag{9}$$

(The change of unit in n' is allowed for in $\delta n'$.)

$$\therefore \delta n = 365{\cdot}25 \times 2\pi x + \delta n' + x(p + u - a \cos \omega_0);$$

$$\therefore \delta n = \frac{\delta n'}{n'} (365{\cdot}25 \times 2\pi + n' + p + u - a \cos \omega_0);$$

$$= \frac{\delta n'}{n'} . n;$$

$$\therefore \frac{\delta n}{n} = \frac{\delta n'}{n'};$$

or the change of unit of time is as

$$1 : 1 + \frac{\delta n'}{n'},$$

as before proved.

Since the epoch from which t is measured is not yet fixed, let it be so chosen that

$$C = L_0.$$

Then

$$s = L_0 + Nt + 365{\cdot}25 \times 2\pi \times t + (b \cos \omega_0 - u')t^2 + \Psi . \cos \omega_0.$$

If the epoch thus chosen be called 1850, Jan. 1, Paris mean noon, we have for the computation of the distance γ from A at the different Paris mean noons,

$$S_0 = L_0 + Nt + (b . \cos \omega_0 - u') . t^2 + \Psi . \cos \omega_0.$$

The non-periodic part of S_0 only differs from the mean longitude of the Sun by

$$\{\lambda - (b . \cos \omega_0 - u')\} . t^2.$$

If, therefore, we call

$$L = L_0 + Nt + (b . \cos \omega_0 - u') . t^2$$

the longitude of the mean Sun, the mean longitude of the Sun

$$= L + \{\lambda - (b . \cos \omega_0 - u')\} . t^2.$$

Here

$$\{\lambda - (b \cos \omega_0 - u')\} . t^2$$

is a small secular term which, with the data adopted in Le Verrier's Tables—

$$= -0{\cdot}00003113 . t^2 = -0{\cdot}3113\left(\frac{t}{100}\right)^2.$$

If it should be found absolutely necessary, in the progress of science, to introduce corrections δN and $\delta\lambda$ to N and λ in order to represent the solar observations, we must keep

$$L = L_0 + Nt + (b \cos \omega_0 - u')t^2$$

still unchanged, but put the mean longitude of the Sun

$$= L + \delta N t + (\delta\lambda - 0{\cdot}00003113) . t^2.$$

If the constants determining the motions of the planes of reference and the mean motion of the Sun are accurately known, we shall, in this way, have for a very long period the point of *reference* γ coincident with the point of intersection of the ecliptic and equator; but even if this condition is not quite satisfied, we must not change γ by changing the law of variation of Aγ until we deliberately change our unit of time, and carry

logically through all our work the consequences of that change. If the difference $0''.3113\left(\frac{t}{100}\right)^2$ between the longitude of the mean Sun and the mean longitude of the Sun be neglected (and this has usually been done), it would lead to the apparent necessity of an increase of about $4''.16$ in the secular acceleration of the Moon's mean motion. This quantity agrees very closely with the supposed discordance between theory and observation. It would appear, therefore, that the discordance between theory and observation which has recently been thrown on the Earth's rotation no longer exists to any proved sensible extent, and, so far as we know at present, the time of the Earth's rotation is constant.

[This abstract was drawn up for the Society's publications by request, after a verbal explanation of the paper, which was given at the meeting of April 13, 1883.]

On the Computation of the Eccentric Anomaly, Equation of the Centre and Radius Vector of a Planet, in Terms of the Mean Anomaly and Eccentricity. By J. Morrison, M.D., M.A., Assistant on the American *Ephemeris and Nautical Almanac*, Washington, D.C.

(*Communicated by the Foreign Secretary.*)

Several methods of computing the eccentric anomaly of a planet from the eccentricity and mean anomaly have already been given, in the *Monthly Notices* and elsewhere, by many very able mathematicians. Some of these methods involve considerable labour even after the formulæ have been deduced, while others give only rough approximations. The following method of treating the subject is due, I believe, to the late Professor Hansen, as indicated by him in the *Abhandlungen der Sächsischen Gesellschaft der Wissenschaften*, Band II. It gives the eccentric anomaly with great accuracy and facility by means of rapidly converging series, and in most cases with very little labour, especially after the coefficients of the several terms of the series have been computed for each planet. I purpose in this short paper to develop the subject more fully than is done in the work referred to, and to apply the results to the computation of the eccentric anomaly in the case of each of the primary planets, as well as of the equation of the centre and the radius vector.

Let

M = the Mean Anomaly,

E = the Eccentric Anomaly,

ν = the True Anomaly,

e = the Eccentricity,

and

ϵ = the Napierian base·

then we have the following well-known relations:

$$\mathrm{M} = \mathrm{E} - e \sin \mathrm{E}; \tag{1}$$

$$\tan \tfrac{1}{2}\nu = \sqrt{\frac{1+e}{1-e}} \cdot \tan \tfrac{1}{2}\mathrm{E}. \tag{2}$$

Hansen assumes

$$y = \epsilon^{\mathrm{E}\sqrt{-1}},$$

and

$$z = \epsilon^{\mathrm{M}\sqrt{-1}};$$

that is,

$$y = \cos \mathrm{E} - \sqrt{-1} \sin \mathrm{E},$$

and

$$z = \cos \mathrm{M} + \sqrt{-1} \sin \mathrm{M},$$

and represents the coefficients of the series to be developed according to the sine and cosine of multiples of E by $\mathrm{P}_h^{(i)}$ and $\mathrm{Q}_i^{(h)}$, thus

$$y^i = \sum_{i=-\infty}^{i=\infty} \mathrm{P}_h^{(i)} z^h \tag{3}$$

$$z^h = \sum_{h=-\infty}^{h=\infty} \mathrm{Q}_i^{(h)} y^i \tag{4}$$

where h and i are *integers*, and it is evident from the form of the series that we must have

$$\mathrm{P}_h^{(i)} = \mathrm{P}_{-h}^{-(i)}$$

and

$$\mathrm{Q}_i^{(h)} = \mathrm{Q}_{-i}^{-(h)},$$

except in the case of $h=0$.

Relation between $\mathrm{P}_h^{(i)}$ *and* $\mathrm{Q}_i^{(h)}$.

Put

$$h = i + \mu \text{ in (3)},$$

and we shall have

$$y^i = \sum_{i=-\infty}^{i=\infty} \mathrm{P}_{i+\mu}^{(i)} z^{i+\mu}.$$

Multiply by $z^{-\mu} d\mathrm{M}$ and integrate between the limits π and $-\pi$, thus

$$\mathrm{P}_{i+\mu}^{(i)} = \frac{1}{2\pi} \int_{-\pi}^{\pi} y^i z^{-\mu-i} d\mathrm{M}$$

$$= \frac{1}{2\pi\sqrt{-1}} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^i z^{-\mu-i-1} dz \tag{5}$$

since

$$dM = \frac{dz}{z\sqrt{-1}}.$$

Again, in (4) put $i = h + \nu$, multiply by $y^{-\nu} dE$, and integrate between the limits π and $-\pi$, thus

$$Q_{h+\nu}^{(h)} = \frac{1}{2\pi} \int_{-\pi}^{\pi} y^{-\nu-h} z^h dE$$

$$= \frac{1}{2\pi\sqrt{-1}} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^{-\nu-h-1} z^h dy \qquad (6)$$

since

$$dE = \frac{dy}{y\sqrt{-1}}.$$

Let

$$i + \mu = h, \text{ and } h + \nu = i;$$

that is

$$\mu + \nu = 0.$$

Then (5) and (6) become

$$P_h^{(i)} = \frac{1}{2\pi\sqrt{-1}} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^i z^{-h-1} dz, \qquad (7)$$

$$Q_i^{(h)} = \frac{1}{2\pi\sqrt{-1}} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^{-i-1} z^h dy. \qquad (8)$$

Integrating (7) by parts, we have,

$$P_h^{(i)} = \frac{1}{2\pi\sqrt{-1}} \cdot \frac{i}{h} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^{i-1} z^{-h} dy, \qquad (9)$$

Comparing (8) and (9), we have

$$P_h^{(i)} = \frac{i}{h} Q_{-i}^{-(h)} = \frac{i}{h} Q_i^{(h)},$$

the required relation.

Eliminating E and M from the equations

$$y = \epsilon^{E\sqrt{-1}}, \quad z = \epsilon^{M\sqrt{-1}}, \text{ and } M = E - e \sin E,$$

we get

$$z = y\, \epsilon^{-\frac{1}{2}e(y-y^{-1})},$$

which, substituted in (8), gives

$$Q_i^{(h)} = \frac{1}{2\pi\sqrt{-1}} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^{h-i-1} \epsilon^{-\frac{1}{2}eh(y-y^{-1})} dy \qquad (10)$$

Put

$$\lambda = -\tfrac{1}{2}eh,$$

and develop

$$\epsilon^{\lambda(y-y^{-1})},$$

by the exponential theorem, thus

$$\begin{aligned}
\epsilon^{\lambda(y-y^{-1})} &= 1 + \lambda(y-y^{-1}) + \frac{\lambda^2}{1.2}(y-y^{-1})^2 + \frac{\lambda^3}{1.2.3}(y-y^{-1})^3 + \text{etc.} \\
&= \left(1 - \frac{\lambda^2}{1^2} + \frac{\lambda^4}{2^2} - \frac{\lambda^6}{6^2} + \frac{\lambda^8}{24^2} - \frac{\lambda^{10}}{120^2} + \ldots\ldots\right) y^0 \\
&\quad + \left(\lambda - \frac{\lambda^3}{1.2} + \frac{\lambda^5}{2\lfloor 3} - \frac{\lambda^7}{6\lfloor 4} + \frac{\lambda^9}{4\lfloor 6} - \ldots\ldots\right) y \\
&\quad + \left(-\lambda + \frac{\lambda^3}{1.2} - \frac{\lambda^5}{2\lfloor 3} + \frac{\lambda^7}{6\lfloor 4} - \frac{\lambda^9}{4\lfloor 6} - \ldots\ldots\right) y^{-1} \\
&\quad + \left(\frac{\lambda^2}{\lfloor 2} - \frac{\lambda^4}{\lfloor 3} + \frac{\lambda^6}{2\lfloor 4} - \frac{\lambda^8}{\lfloor 6} + \frac{\lambda^{10}}{24\lfloor 6} - \ldots\ldots\right) y^2 \\
&\quad + \left(\frac{\lambda^2}{\lfloor 2} - \frac{\lambda^4}{\lfloor 3} + \frac{\lambda^6}{2\lfloor 4} - \frac{\lambda^8}{\lfloor 6} + \frac{\lambda^{10}}{24\lfloor 6} - \ldots\ldots\right) y^{-2} \\
&\quad + \left(\frac{\lambda^3}{\lfloor 3} - \frac{\lambda^5}{\lfloor 4} + \frac{\lambda^7}{2\lfloor 5} - \frac{\lambda^9}{6\lfloor 6} + \frac{\lambda^{11}}{24\lfloor 7} - \ldots\ldots\right) y^3 \\
&\quad + \left(-\frac{\lambda^3}{\lfloor 3} + \frac{\lambda^5}{\lfloor 4} - \frac{\lambda^7}{2\lfloor 5} + \frac{\lambda^9}{6\lfloor 6} - \frac{\lambda^{11}}{24\lfloor 7} + \ldots\ldots\right) y^{-3} \\
&\quad + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\quad + \frac{\lambda^m}{\lfloor m}\left(1 - \frac{\lambda^2}{1.m+1} + \frac{\lambda^4}{1.2.m+1.m+2}\right. \\
&\qquad \left. - \frac{\lambda^6}{1.2.3.m+1.m+2.m+3} + \ldots\right) y^m
\end{aligned}$$

where m must be considered a positive integer.

Let us now represent the coefficient of y^m by $J_\lambda^{(m)}$ (Hansen's notation for the Besselian Function); then the above series may be written thus:

$$\epsilon^{\lambda(y-y^{-1})} = \sum_{m=-\infty}^{m=\infty} J_\lambda^{(m)} y^m, \qquad (11)$$

and from the form of the preceding development we notice that

$$J_\lambda^{(-m)} = (-1)^m J_\lambda^{(m)}, \quad J_{-\lambda}^{(m)} = (-1)^m J_\lambda^{(m)}, \quad J_{-\lambda}^{(-m)} = J_\lambda^{(m)}.$$

From the last two equations we have,

$$Q_i^{(h)} = \frac{1}{2\pi\sqrt{-1}} \sum_{m=-\infty}^{m=\infty} J_\lambda^{(m)} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^{h-i-m-1} dy. \qquad (12)$$

Now, since

$$\int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^n dy = 0,$$

when n is an integer and

$$\int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^{-1} dy = 2\pi\sqrt{-1},$$

we must have $h-i-m=0$, otherwise (12) would vanish; therefore we have

$$Q_i^{(h)} = \frac{1}{2\pi\sqrt{-1}} \sum_{m=-\infty}^{m=\infty} J_\lambda^{(m)} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^{-1} dy$$

$$= \sum_{m=-\infty}^{m=\infty} J_\lambda^{(m)};$$

hence

$$P_h^{(i)} = \frac{i}{h} \sum_{m=-\infty}^{m=\infty} J_\lambda^{(m)}.$$

But

$$y^i = \sum_{i=-\infty}^{i=\infty} P_h^{(i)} z^h = \frac{i}{h} \sum_{h=-\infty}^{h=\infty} J_\lambda^{(h-i)} z^h;$$

and also

$$y^i = \cos i\mathrm{E} + \sqrt{-1} \sin i\mathrm{E}$$

$$z^h = \cos h\mathrm{M} + \sqrt{-1} \sin h\mathrm{M};$$

therefore we have

$$\cos i\mathrm{E} + \sqrt{-1} \sin i\mathrm{E} = \frac{i}{h} \sum_{h=-\infty}^{h=\infty} J_\lambda^{(h-i)} (\cos h\mathrm{M} + \sqrt{-1} \sin h\mathrm{M}).$$

Equating the real and imaginary parts of this equation we have

$$\cos i\mathrm{E} = \frac{i}{h} \sum_{h=-\infty}^{h=\infty} J_\lambda^{(h-i)} \cos h\mathrm{M} \qquad (13)$$

$$\sin i\mathrm{E} = \frac{i}{h} \sum_{h=-\infty}^{h=\infty} J_\lambda^{(h-i)} \sin h\mathrm{M} \qquad (14)$$

When $h=0$, the second members of the last two equations assume the form $\frac{0}{0}$; hence it becomes necessary to evaluate them. Putting $h=0$ in (7) it becomes

$$P_0^{(i)} = \frac{1}{2\pi\sqrt{-1}} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^i z^{-1}\, dz.$$

Differentiating

$$z = y\epsilon^{-\frac{1}{2}e\,(y-y^{-1})}$$

we get

$$z^{-1}\, dz = y^{-1}\, dy - \frac{e}{2}\,(1+y^{-2})\, dy,$$

which, substituted in the preceding equation, gives

$$P_0^{(i)} = \frac{1}{2\pi\sqrt{-1}} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} (y^{i-1} - \frac{e}{2}\, y^i - \frac{e}{2}\, y^{i-2})\, dy$$

$$= 0,$$

for all values of i greater than 1 or less than -1. Therefore, 0, 1 and -1 are the values of i, which make the second members of (13) and (14) take the form $\frac{0}{0}$.

When $i=0$, we have

$$P_0^{(0)} = \frac{1}{2\pi\sqrt{-1}} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} (y^{-1} - \frac{e}{2} - \frac{e}{2}\, y^{-2})\, dy$$

$$= \frac{1}{2\pi\sqrt{-1}} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} y^{-1}\, dy - \frac{1}{2\pi\sqrt{-1}} \cdot \frac{e}{2} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} (1+y^{-2})\, dy$$

$$= 1 - 0 = 1.$$

When $i=1$, we have

$$P_0^{(i)} = \frac{1}{2\pi\sqrt{-1}} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} (1 - \frac{e}{2}\, y - \frac{e}{2}\, y^{-1})\, dy = \tfrac{1}{2}e.$$

When $i=-1$, we have

$$P_0^{(-1)} = \frac{1}{2\pi\sqrt{-1}} \int_{\epsilon^{-\pi\sqrt{-1}}}^{\epsilon^{\pi\sqrt{-1}}} (y^{-2} - \frac{e}{2}\, y^{-1} - \frac{e}{2}\, y^{-3})\, dy = -\tfrac{1}{2}e.$$

For all other values of i we have

$$P_0^{(i)} = 0.$$

From (13) we find, when $i=1$, 2, &c.

$$\cos E = -\frac{e}{2} + \left(1 - \frac{3e^2}{2^3} + \frac{5e^4}{2^6\cdot 3} - \frac{7e^6}{2^{10}\cdot 3^2} + \frac{e^8}{2^{14}\cdot 5} - \frac{11e^{10}}{2^{17}\cdot 3^3\cdot 5^2} + \ldots\right)\cos M$$

$$+ \left(\frac{e}{2} - \frac{e^3}{3} + \frac{e^5}{2^4} - \frac{e^7}{2^2\cdot 3^2\cdot 5} + \frac{e^9}{2^7\cdot 3^3} - \frac{e^{11}}{2^6\cdot 3^2\cdot 5^2\cdot 7} + \ldots\right)\cos 2M$$

$$+ \left(\frac{3e^2}{2^3} - \frac{3^2\cdot 5e^4}{2^7} + \frac{7\cdot 3^4e^6}{2^{10}\cdot 5} - \frac{3^6e^8}{2^{13}\cdot 5} + \frac{3^6\cdot 11e^{10}}{2^{17}\cdot 5\cdot 7} - \ldots\ldots\ldots\right)\cos 3M$$

$$+ \left(\frac{e^3}{3} - \frac{2e^5}{5} + \frac{2^3e^7}{3^2\cdot 5} - \frac{2^3e^9}{3^3\cdot 7} + \frac{2e^{11}}{3^2\cdot 5\cdot 7} - \ldots\ldots\ldots\right)\cos 4M$$

$$+ \left(\frac{5^3e^4}{2^7\cdot 3} - \frac{5^4\cdot 7e^6}{2^{10}\cdot 3^2} + \frac{5^6e^8}{2^{13}\cdot 7} - \frac{5^8\cdot 11e^{10}}{2^{18}\cdot 3^3\cdot 7} + \ldots\ldots\ldots\ldots\right)\cos 5M$$

$$+ \left(\frac{3^3e^5}{2^4\cdot 5} - \frac{3^4e^7}{2^2\cdot 5\cdot 7} + \frac{3^6e^9}{2^8\cdot 7} - \frac{3^6e^{11}}{2^7\cdot 5\cdot 7} + \ldots\ldots\ldots\ldots\right)\cos 6M$$

$$+ \left(\frac{7^5e^6}{2^{10}\cdot 3^2\cdot 5} - \frac{7^6e^8}{2^{15}\cdot 5} + \frac{7^9\cdot 11e^{10}}{2^{18}\cdot 3^4\cdot 5\cdot 7} - \ldots\ldots\ldots\ldots\right)\cos 7M$$

$$+ \left(\frac{2^7e^7}{3^2\cdot 5\cdot 7} - \frac{2^9e^9}{3^4\cdot 7} + \frac{2^{12}e^{11}}{3^3\cdot 5^2\cdot 7} - \ldots\ldots\ldots\ldots\right)\cos 8M$$

$$+ \left(\frac{3^{12}e^8}{2^{15}\cdot 5\cdot 7} - \frac{3^{14}\cdot 11e^{10}}{2^{18}\cdot 5^2\cdot 7} + \ldots\ldots\ldots\ldots\right)\cos 9M$$

$$+ \left(\frac{5^7e^9}{2^8\cdot 3^4\cdot 7} - \frac{5^8e^{11}}{2^7\cdot 3^3\cdot 7\cdot 11} + \ldots\ldots\ldots\ldots\right)\cos 10M$$

$$+ \left(\frac{11^9e^{10}}{2^{18}\cdot 3^4\cdot 5^2\cdot 7} - \ldots\ldots\ldots\ldots\right)\cos 11M$$

$$+ \left(\frac{2\cdot 3^6e^{11}}{5^2\cdot 7\cdot 11} - \ldots\ldots\ldots\ldots\right)\cos 12M$$

$$+ \ldots\ldots\ldots$$

A check in this and the next series is obtained by putting M=o when the second member reduces to 1 as it should.

$$
\begin{aligned}
\cos 2E = {} & \Big(&& - e && + \frac{e^3}{2^2\cdot 3} && - \frac{e^5}{2^7\cdot 3} && + \frac{e^7}{2^9\cdot 3^2\cdot 5} && - \frac{e^9}{2^{14}\cdot 3^3\cdot 5} && + \ldots \Big) \cos M \\
& + \Big(1 && - e^2 && + \frac{5e^4}{2^3\cdot 3} && - \frac{7e^6}{2^3\cdot 3^2\cdot 5} && + \frac{e^8}{2^6\cdot 3\cdot 5} && - \frac{11e^{10}}{2^8\cdot 3^3\cdot 5^2\cdot 7} && + \ldots \Big) \cos 2M \\
& + \Big(e && - \frac{3^2e^3}{2^3} && + \frac{3^5e^5}{2^7\cdot 5} && - \frac{3^4e^7}{2^8\cdot 5} && + \frac{3^6e^9}{2^{14}\cdot 7} && - \ldots\ldots\ldots \Big) \cos 3M \\
& + \Big(e^2 && - \frac{2^2e^4}{3} && + \frac{2^7\cdot 7e^6}{3^2\cdot 5} && - \frac{2^4e^8}{3\cdot 5\cdot 7} && + \frac{2\cdot 11e^{10}}{3^3\cdot 5\cdot 7} && - \ldots\ldots\ldots \Big) \cos 4M \\
& + \Big(\frac{5^2e^3}{2^3\cdot 3} && - \frac{5^4e^5}{2^7\cdot 3} && + \frac{5^6e^7}{2^8\cdot 3^2\cdot 7} && - \frac{5^9e^9}{2^{15}\cdot 3^3\cdot 7} && + \ldots\ldots\ldots \Big) \cos 5M \\
& + \Big(\frac{3^2e^4}{2^3} && - \frac{3^4e^6}{2^3\cdot 5} && + \frac{3^8e^8}{2^7\cdot 5\cdot 7} && - \frac{3^5\cdot 11e^{10}}{2^7\cdot 5\cdot 7} && + \ldots\ldots\ldots \Big) \cos 6M \\
& + \Big(\frac{7^4e^5}{2^7\cdot 3\cdot 5} && - \frac{7^6e^7}{2^{10}\cdot 3^2\cdot 5} && + \frac{7^8e^9}{2^{15}\cdot 3^4} && - \ldots\ldots\ldots \Big) \cos 7M \\
& + \Big(\frac{2^6e^6}{3^2\cdot 5} && - \frac{2^{10}e^8}{3^2\cdot 5\cdot 7} && + \frac{2^{12}\cdot 11e^{10}}{3^4\cdot 5^2\cdot 7} && - \ldots\ldots\ldots \Big) \cos 8M \\
& + \Big(\frac{3^{10}e^7}{2^{10}\cdot 5\cdot 7} && - \frac{3^{14}e^9}{2^{15}\cdot 5\cdot 7} && + \ldots\ldots\ldots \Big) \cos 9M \\
& + \Big(\frac{5^6e^8}{2^7\cdot 3^2\cdot 7} && - \frac{5^8e^{10}}{2^7\cdot 3^4\cdot 7} && + \ldots\ldots\ldots \Big) \cos 10M \\
& + \Big(\frac{11^8e^9}{2^{15}\cdot 3^4\cdot 5\cdot 7} && - \ldots\ldots\ldots \Big) \cos 11M \\
& + \Big(\frac{2\cdot 3^5e^{10}}{5^2\cdot 7} && - \ldots\ldots\ldots \Big) \cos 12M \\
& + \ldots\ldots\ldots
\end{aligned}
$$

and so on.

Putting $i=1$ in (14), multiplying the resulting series by e, and substituting in (1), we have the following converging series for the direct computation of E.

$$
\begin{aligned}
E = M + \Big(& e - \frac{e^3}{2^3} + \frac{e^5}{2^6 \cdot 3} - \frac{e^7}{2^{10} \cdot 3^2} + \frac{e^9}{2^{13} \cdot 3^2 \cdot 5} - \frac{e^{11}}{2^{17} \cdot 3^3 \cdot 5^2} + \ldots \Big) \sin M \\
+ \Big(& \frac{e^2}{2} - \frac{e^4}{2 \cdot 3} + \frac{e^6}{2^4 \cdot 3} - \frac{e^8}{2^4 \cdot 3^2 \cdot 5} + \frac{e^{10}}{2^7 \cdot 3^3 \cdot 5} - \frac{e^{12}}{2^7 \cdot 3^3 \cdot 5^2 \cdot 7} + \ldots \Big) \sin 2M \\
+ \Big(& \frac{3e^3}{2^3} - \frac{3^3 e^5}{2^7} + \frac{3^5 e^7}{2^{10} \cdot 5} - \frac{3^5 e^9}{2^{13} \cdot 5} + \frac{3^7 e^{11}}{2^7 \cdot 5 \cdot 7} - \ldots \ldots \Big) \sin 3M \\
+ \Big(& \frac{e^4}{3} - \frac{2^2 e^6}{3 \cdot 5} + \frac{2^2 e^8}{3^2 \cdot 5} - \frac{2^4 e^{10}}{3^3 \cdot 5 \cdot 7} + \frac{2 e^{12}}{3^3 \cdot 5 \cdot 7} - \ldots \ldots \Big) \sin 4M \\
+ \Big(& \frac{5^3 e^5}{2^7 \cdot 3} - \frac{5^5 e^7}{2^{10} 3^2} + \frac{5^7 e^9}{2^{12} \cdot 3^2 \cdot 7} - \frac{5^9 e^{11}}{2^{18} \cdot 3^3 \cdot 7} + \ldots \ldots \Big) \sin 5M \\
+ \Big(& \frac{3^3 e^6}{2^4 \cdot 5} - \frac{3^5 e^8}{2^4 \cdot 5 \cdot 7} + \frac{3^7 e^{10}}{2^8 \cdot 5 \cdot 7} - \frac{3^6 e^{12}}{2^8 \cdot 5 \cdot 7} + \ldots \ldots \Big) \sin 6M \\
+ \Big(& \frac{7^5 e^7}{2^{10} \cdot 3^2 \cdot 5} - \frac{7^7 e^9}{2^{15} \cdot 3^2 \cdot 5} + \frac{7^9 e^{11}}{2^{18} \cdot 3^4 \cdot 5} - \ldots \ldots \Big) \sin 7M \\
+ \Big(& \frac{2^7 e^8}{3^2 \cdot 5 \cdot 7} - \frac{2^{11} e^{10}}{3^4 \cdot 5 \cdot 7} + \frac{2^{13} e^{12}}{3^4 \cdot 5^2 \cdot 7} - \ldots \ldots \Big) \sin 8M \\
+ \Big(& \frac{3^{12} e^9}{2^{15} \cdot 5 \cdot 7} - \frac{3^{16} e^{11}}{2^{18} \cdot 5^2 \cdot 7} + \ldots \ldots \Big) \sin 9M \\
+ \Big(& \frac{5^7 e^{10}}{2^8 \cdot 3^4 \cdot 7} - \frac{5^9 e^{12}}{2^8 \cdot 3^4 \cdot 7 \cdot 11} + \ldots \ldots \Big) \sin 10M \\
+ \Big(& \frac{11^9 e^{11}}{2^{18} \cdot 3^4 \cdot 5^2 \cdot 7} - \ldots \ldots \Big) \sin 11M \\
+ \Big(& \frac{2 \cdot 3^6 e^{12}}{5^2 \cdot 7 \cdot 11} - \ldots \ldots \Big) \sin 12M \\
+ & \ldots \ldots
\end{aligned}
$$

Reducing the coefficients of sin M, sin 2M, &c., to logarithms, we have the following series for each of the primary planets.

Mercury.

$e = \cdot 20560478$ $\qquad e'' = 42409''\cdot 03$

$$\begin{aligned} E = M &+ [4\cdot 6251614] \sin M + [3\cdot 6333310] \sin 2M \\ &+ [2\cdot 8172458] \sin 3M + [2\cdot 0747079] \sin 4M \\ &+ [1\cdot 3733623] \sin 5M + [0\cdot 6972126] \sin 6M \\ &+ [0\cdot 0394355] \sin 7M + [9\cdot 3948531] \sin 8M \\ &+ [8\cdot 7587952] \sin 9M + [8\cdot 1318557] \sin 10M \\ &+ [7\cdot 5113252] \sin 11M + \end{aligned}$$

For finding the true anomaly we shall have

$$\tan \tfrac{1}{2}\nu = [0\cdot 0905841] \tan \tfrac{1}{2} E.$$

Example. Let $M = 64^\circ\ 10'$.

$$\begin{aligned} E = 64^\circ\ 10' &+ 37969''\cdot 5526 + 3371''\cdot 9202 - 142''\cdot 0962 \\ &- 115''\cdot 5688 - 14''\cdot 9202 + 2''\cdot 1046 + 1''\cdot 0950 \\ &+ 0''\cdot 1114 - 0''\cdot 0349 - 0''\cdot 0132 - 0''\cdot 0008 \\ = 64^\circ\ 10' &+ 11^\circ\ 24'\ 32''\cdot 150 = 75^\circ\ 34'\ 32''\cdot 150 \\ \nu = 87^\circ\ 22'\ 20''\cdot 29. & \end{aligned}$$

Venus.

$e = \cdot 00684331$ $\qquad e'' = 1411''\cdot 534$

$$\begin{aligned} E = M &+ [3\cdot 1496888] \sin M + [0\cdot 6839207] \sin 2M \\ &+ [8\cdot 3942434] \sin 3M + [6\cdot 1783520] \sin 4M \end{aligned}$$

$$\tan \tfrac{1}{2}\nu = [0\cdot 0029720] \tan \tfrac{1}{2} E.$$

The Earth.

$e = \cdot 0167711$ $\qquad e'' = 3459''\cdot 287$

$$\begin{aligned} E = M &+ [3\cdot 5389714] \sin M + [1\cdot 4624774] \sin 2M \\ &+ [9\cdot 5620725] \sin 3M + [7\cdot 7354525] \sin 4M \end{aligned}$$

$$\tan \tfrac{1}{2}\nu = [0\cdot 0072842] \tan \tfrac{1}{2} E.$$

Example. Let $M = 71^\circ$.

$$\begin{aligned} E = 71^\circ &+ 54'\ 30''\cdot 7060 + 17''\cdot 8574 - 0''\cdot 1987 - 0''\cdot 00527 \\ = 71^\circ\ 54'\ 48''\cdot 359. & \end{aligned}$$

Check.

$$E = M + e'' \sin E$$
$$= 71^\circ + 3288''\cdot359 = 71^\circ\ 54'\ 48''\cdot359.$$

Mars.

$e = \cdot09326113$ $\qquad e'' = 19236''\cdot5$

$$E = M + [4\cdot2836536] \sin M + [2\cdot9515369] \sin 2M$$
$$+ [1\cdot7954344] \sin 3M + [0\cdot7130829] \sin 4M$$
$$+ [9\cdot6715855] \sin 5M + [8\cdot6560435] \sin 6M$$
$$+ [7\cdot6585193] \sin 7M +$$
$$\tan \tfrac{1}{2}\nu = [0\cdot0406208] \tan \tfrac{1}{2}E.$$

Jupiter.

$e = \cdot0482519$ $\qquad e'' = 9952''\cdot67$

$$E = M + [3\cdot9978131] \sin M + [2\cdot3800869] \sin 2M$$
$$+ [0\cdot9384309] \sin 3M + [9\cdot5706569] \sin 4M$$
$$+ [8\cdot2435235] \sin 5M + [6\cdot9424833] \sin 6M$$
$$\tan \tfrac{1}{2}\nu = [0\cdot0209718] \tan \tfrac{1}{2}E.$$

Saturn.

$e = \cdot0559428$ $\qquad e'' = 11539''\cdot02$

$$E = M + [4\cdot0619992] \sin M + [2\cdot5084302] \sin 2M$$
$$+ [1\cdot1309242] \sin 3M + [9\cdot8271927] \sin 4M$$
$$+ [8\cdot5643061] \sin 5M + [7\cdot3274187] \sin 6M$$
$$\tan \tfrac{1}{2}\nu = [0\cdot0243210] \tan \tfrac{1}{2}E.$$

Uranus.

$e = \cdot0463592$ $\qquad e'' = 9562''\cdot29$

$$E = M + [3\cdot9804453] \sin M + [2\cdot3453568] \sin 2M$$
$$+ [0\cdot8917040] \sin 3M + [9\cdot5011044] \sin 4M$$
$$+ [8\cdot1567170] \sin 5M + [6\cdot8383189] \sin 6M$$
$$\tan \tfrac{1}{2}\nu = [0\cdot0201480] \tan \tfrac{1}{2}E.$$

Neptune.

$e = \cdot0089903$ $\qquad e'' = 1854''\cdot38$

$$E = M + [3\cdot2681948] \sin M + [0\cdot9209318] \sin 2M$$
$$+ [8\cdot7497592] \sin 3M + [6\cdot6523734] \sin 4M$$
$$\tan \tfrac{1}{2}\nu = [0\cdot0039045] \tan \tfrac{1}{2}E.$$

Equation of the Centre.

From the theory of elliptic motion we have

$$\frac{dv}{dt} = \frac{h}{r^2},$$

where h=half the area described in a unit of time; therefore

$$\frac{\mathrm{T}}{\pi ab} = \frac{2}{h};$$

whence

$$h = \frac{2\pi ab}{\mathrm{T}};$$

therefore

$$\frac{dv}{dt} = \frac{2\pi ab}{\mathrm{T}r^2},$$

and

$$\frac{d\mathrm{M}}{dt} = \frac{2\pi}{\mathrm{T}}.$$

Now,

$$\frac{dv}{d\mathrm{M}} = \frac{dv}{dt} \cdot \frac{dt}{d\mathrm{M}} = \frac{ab}{r^2};$$

but

$$b = a\sqrt{1-e^2},$$

and

$$r = a(1-e\cos \mathrm{E});$$

therefore

$$\frac{dv}{d\mathrm{M}} = \frac{\sqrt{1-e^2}}{(1-e\cos \mathrm{E})^2} \qquad (15)$$

Developing the second member by the binomial theorem, and changing the powers of cos E to the cosines of multiples of E, we have

$$\frac{dv}{d\mathrm{M}} = (1-e^2)^{\frac{1}{2}}\,(1-e\cos \mathrm{E})^{-2}$$

$$= (1 + e^2 + e^4 + e^6 + e^8 + e^{10} + e^{12} + \ldots)$$
$$+ 2(e + e^3 + e^5 + e^7 + e^9 + e^{11} + \ldots)\cos \mathrm{E}$$
$$+ \left(\frac{3}{2}e^2 + \frac{7}{2^2}e^4 + \frac{59}{2^5}e^6 + \frac{121}{2^6}e^8 + \frac{491}{2^8}e^{10} + \frac{991}{2^9}e^{12} + \ldots\right)\cos 2\,\mathrm{E}$$
$$+ \left(e^3 + \frac{11}{2^3}e^5 + \frac{25}{2^4}e^7 + \frac{107}{2^6}e^9 + \frac{223}{2^7}e^{11} + \ldots\right)\cos 3\,\mathrm{E}$$
$$+ \left(\frac{5}{2^3}e^4 + e^6 + \frac{79}{2^6}e^8 + \frac{89}{2^6}e^{10} + \frac{3073}{2^{11}}e^{12} + \ldots\right)\cos 4\,\mathrm{E}$$
$$+ \left(\frac{3}{2^3}e^5 + \frac{11}{2^4}e^7 + \frac{59}{2^6}e^9 + \frac{281}{2^8}e^{11} + \ldots\right)\cos 5\,\mathrm{E}$$
$$+ \left(\frac{7}{2^5}e^6 + \frac{29}{2^6}e^8 + \frac{337}{2^9}e^{10} + \frac{849}{2^{10}}e^{12} + \ldots\right)\cos 6\,\mathrm{E}$$
$$+ \left(\frac{1}{2^3}e^7 + \frac{37}{2^7}e^9 + \frac{29}{2^6}e^{11} + \ldots\right)\cos 7\,\mathrm{E}$$
$$+ \left(\frac{9}{2^7}e^8 + \frac{23}{2^7}e^{10} + \frac{155}{2^9}e^{12} + \ldots\right)\cos 8\,\mathrm{E}$$
$$+ \left(\frac{5}{2^7}e^9 + \frac{7}{2^6}e^{11} + \ldots\right)\cos 9\,\mathrm{E}$$
$$+ \left(\frac{11}{2^9}e^{10} + \frac{121}{2^{11}}e^{12} + \ldots\right)\cos 10\,\mathrm{E}$$
$$+ \left(\frac{3}{2^8}e^{11} + \ldots\right)\cos 11\,\mathrm{E}$$
$$+ \left(\frac{13}{2^{11}}e^{12} + \ldots\right)\cos 12\,\mathrm{E}$$

Substituting the values of cos E, cos 2E, &c., obtained from (13), multiplying by dM and integrating, we have, after putting C, the equation of the centre, $=\nu - M$

$$
\begin{aligned}
C = {} & + \left(2e - \frac{e^3}{2^2} + \frac{5e^5}{2^5 \cdot 3} + \frac{107e^7}{2^9 \cdot 3^2} + \frac{6217e^9}{2^{13} \cdot 3^2 \cdot 5} + \frac{565879e^{11}}{2^{16} \cdot 3^3 \cdot 5^2} + \ldots \right) \sin M \\
& + \left(\frac{5e^2}{2^2} - \frac{11e^4}{2^3 \cdot 3} + \frac{17e^6}{2^6 \cdot 3} + \frac{43e^8}{2^7 \cdot 3^2 \cdot 5} + \frac{677e^{10}}{2^9 \cdot 3^3 \cdot 5} + \frac{7237e^{12}}{2^{10} \cdot 3^3 \cdot 5 \cdot 7} + \ldots \right) \sin 2M \\
& + \left(\frac{13e^3}{2^2 \cdot 3} - \frac{43e^5}{2^6} + \frac{95e^7}{2^9} - \frac{973e^9}{2^{12} \cdot 3 \cdot 5} + \frac{19503e^{11}}{2^{16} \cdot 5 \cdot 7} + \ldots \ldots \ldots \right) \sin 3M \\
& + \left(\frac{103e^4}{2^5 \cdot 3} - \frac{451e^6}{2^5 \cdot 3 \cdot 5} + \frac{4123e^8}{2^8 \cdot 3^2 \cdot 5} - \frac{1619e^{10}}{2^7 \cdot 3^3 \cdot 7} + \frac{111929e^{12}}{2^{13} \cdot 3^3 \cdot 5 \cdot 7} + \ldots \ldots \ldots \right) \sin 4M \\
& + \left(\frac{1097e^5}{2^6 \cdot 3 \cdot 5} - \frac{5957e^7}{2^9 \cdot 3^2} + \frac{164921e^9}{2^{12} \cdot 3^2 \cdot 7} - \frac{4305913e^{11}}{2^{17} \cdot 3^3 \cdot 7} + \ldots \ldots \ldots \ldots \right) \sin 5M \\
& + \left(\frac{1223e^6}{2^6 \cdot 3 \cdot 5} - \frac{7913e^8}{2^7 \cdot 5 \cdot 7} + \frac{7751e^{10}}{2^{10} \cdot 7} - \frac{82021e^{12}}{2^{11} \cdot 3 \cdot 5 \cdot 7} + \ldots \ldots \ldots \ldots \right) \sin 6M \\
& + \left(\frac{47273e^7}{2^9 \cdot 3^2 \cdot 7} - \frac{1773271e^9}{2^{14} \cdot 3^2 \cdot 5} + \frac{93521303e^{11}}{2^{17} \cdot 3^4 \cdot 5} - \ldots \ldots \ldots \ldots \ldots \right) \sin 7M \\
& + \left(\frac{556403e^8}{2^{10} \cdot 3^2 \cdot 5 \cdot 7} - \frac{1182827e^{10}}{2^7 \cdot 3^4 \cdot 5 \cdot 7} + \frac{32431949e^{12}}{2^{12} \cdot 3^4 \cdot 5 \cdot 7} - \ldots \ldots \ldots \ldots \ldots \right) \sin 8M \\
& + \left(\frac{10661993e^9}{2^{14} \cdot 3^2 \cdot 5 \cdot 7} - \frac{101836961e^{11}}{2^{17} \cdot 5^2 \cdot 7} + \ldots \ldots \ldots \ldots \ldots \ldots \ldots \right) \sin 9M \\
& + \left(\frac{72815870^{10}}{2^{10} \cdot 3^4 \cdot 5 \cdot 7} - \frac{769805651e^{12}}{2^{12} \cdot 3^4 \cdot 5 \cdot 7 \cdot 11} + \ldots \ldots \ldots \ldots \ldots \ldots \ldots \right) \sin 10M \\
& + \left(\frac{6292901710 1e^{11}}{2^{17} \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11} - \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \right) \sin 11M \\
& + \left(\frac{7218065e^{12}}{2^{13} \cdot 3 \cdot 7 \cdot 11} - \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \right) \sin 12M
\end{aligned}
$$

Greatest Equation of the Centre.

When the equation of the centre has its maximum value, $d(\nu - \mathrm{M}) = 0$ or $d\nu = d\mathrm{M}$; therefore, from (15), we have

$$\cos \mathrm{E} = \frac{1-(1-e^2)^{\frac{1}{2}}}{e}$$

$$= \frac{e}{2^2} + \frac{3e^3}{2^5} + \frac{7e^5}{2^7} + \frac{77e^7}{2^{11}} + \ldots$$

and

$$\sin \mathrm{E} = 1 - \frac{e^2}{2^5} - \frac{49e^4}{2^{11}} - \frac{1233e^6}{2^{16}} - \ldots \qquad (16)$$

whence

$$\sin 2\,\mathrm{E} = \frac{e}{2} + \frac{11e^3}{2^6} + \frac{375e^5}{2^{12}} + \frac{7587e^7}{2^{17}} + \ldots$$

$$\sin 3\,\mathrm{E} = -1 + \frac{9e^2}{2^5} + \frac{417e^4}{2^{11}} + \frac{10087e^6}{2^{13}} + \ldots$$

&c., &c.

Any one of these will give the eccentric anomaly when the equation of the centre is a maximum. Developing

$$\tan \tfrac{1}{2}\nu = \sqrt{\frac{1+e}{1-e}} \cdot \tan \tfrac{1}{2}\,\mathrm{E}$$

into a series, we have

$$\nu = \mathrm{E} + 2 \tan \frac{\phi}{2} \sin \mathrm{E} + \tan^2 \frac{\phi}{2} \sin 2\mathrm{E} + \frac{2}{3} \tan^3 \frac{\phi}{2} \sin 3\mathrm{E} + \text{ &c.},$$

where

$$\tan \frac{\phi}{2} = \frac{1-(1-e^2)^{\frac{1}{2}}}{e}$$

$$= \frac{e}{2} + \frac{e^3}{2^3} + \frac{e^5}{2^4} + \frac{5e^7}{2^7} + \text{ &c.}$$

Substituting the values of sin E, sin 2E, &c., $\tan \frac{\phi}{2}$, $\tan^2 \frac{\phi}{2}$ &c., we find

$$\nu = \mathrm{E} + e + \frac{25}{2^5 \cdot 3} e^3 + \frac{1443}{2^{11} \cdot 5} e^5 + \frac{43069}{2^{16} \cdot 7} e^7 + \ldots \qquad (17)$$

Multiplying the second series of (16) by e, and substituting in (1), we get

$$M = E - e + \frac{e^3}{2^3} + \frac{49}{2^{11}}e^5 + \frac{1233}{2^{16}}e^7 + \ldots \quad (18)$$

Subtracting (18) from (17), we have

$$C = \nu - M$$
$$= 2e + \frac{11}{2^4 \cdot 3}e^3 + \frac{599}{2^{10} \cdot 5}e^5 + \frac{17219}{2^{15} \cdot 7}e^7 + \ldots \quad (19)^*$$

Reverting this series we have

$$e = \frac{C}{2} - \frac{11}{2^8 \cdot 3}C^3 - \frac{587}{2^{16} \cdot 3 \cdot 5}C^5 - \frac{40583}{2^{23} \cdot 3^2 \cdot 5 \cdot 7}C^7 - \ldots \quad (20)$$

The Radius Vector.

By the properties of the ellipse we have $r = a(1 - e \cos E)$, in which substitute the values of cos E; we thus deduce the following series, a check upon which is obtained by putting M=0, when it reduces to $\frac{r}{a} = 1 - e$.

---

* Delambre, in his *Astronomy*, vol. ii. p. 55, has given this series correctly, but farther down on the same page he committed an error in reducing the term in e^7, which he gave as $\frac{17219}{2^{16} \cdot 7}e^7$. He also gave the second term of series (20 incorrectly as $-\frac{11}{2^4 \cdot 3}C^3$. Both of these incorrect values were copied by Baily in his "Astronomical Tables and Formulæ," and from him by several other writers on Astronomy. Mr. Farrel in the *Monthly Notices* of the Royal Astronomical Society, June 1860, corrects Baily as to series (20), but at the same time adopts Delambre's incorrect value of the last term of (19), and computes accordingly.

$$\frac{r}{a} = +\left(1 + \frac{e^2}{2}\right)$$

$$+\left(-e + \frac{3}{2^3}e^3 - \frac{5}{2^6\cdot 3}e^5 + \frac{7}{2^{10}\cdot 3^2}e^7 - \frac{1}{2^{14}\cdot 5}e^9 + \frac{11}{2^{17}\cdot 3^3\cdot 5^2}e^{11} - \ldots\right)\cos M$$

$$+\left(-\frac{e^2}{2} + \frac{1}{3}e^4 - \frac{1}{2^4}e^6 + \frac{1}{2^2\cdot 3^2\cdot 5}e^8 - \frac{1}{2^7\cdot 3^3}e^{10} + \frac{1}{2^6\cdot 3^2\cdot 5^2\cdot 7}e^{12} - \ldots\right)\cos 2M$$

$$+\left(-\frac{3}{2^3}e^3 + \frac{3^2\cdot 5}{2^7}e^5 - \frac{7\cdot 3^4}{2^{10}\cdot 5}e^7 + \frac{3^6}{2^{13}\cdot 5}e^9 - \frac{3^6\cdot 11}{2^{17}\cdot 5\cdot 7}e^{11} + \ldots\right)\cos 3M$$

$$+\left(-\frac{1}{3}e^4 + \frac{2}{5}e^6 - \frac{2^3}{3^2\cdot 5}e^8 + \frac{2^3}{3^3\cdot 7}e^{10} - \frac{2}{3^2\cdot 5\cdot 7}e^{12} + \ldots\right)\cos 4M$$

$$+\left(-\frac{5^3}{2^7\cdot 3}e^5 + \frac{5^4\cdot 7}{2^{10}\cdot 3^2}e^7 - \frac{5^6}{2^{13}\cdot 7}e^9 + \frac{5^8\cdot 11}{2^{18}\cdot 3^3\cdot 7}e^{11} - \ldots\right)\cos 5M$$

$$+\left(-\frac{3^3}{2^4\cdot 5}e^6 + \frac{3^4}{2^2\cdot 5\cdot 7}e^8 - \frac{3^6}{2^8\cdot 7}e^{10} + \frac{3^6}{2^7\cdot 5\cdot 7}e^{12} - \ldots\right)\cos 6M$$

$$+\left(-\frac{7^5}{2^{10}\cdot 3^2\cdot 5}e^7 + \frac{7^6}{2^{15}\cdot 5}e^9 - \frac{7^9\cdot 11}{2^{18}\cdot 3^4\cdot 5\cdot 7}e^{11} + \ldots\right)\cos 7M$$

$$+\left(-\frac{2^7}{3^2\cdot 5\cdot 7}e^8 + \frac{2^9}{3^4\cdot 7}e^{10} - \frac{2^{12}}{3^3\cdot 5^2\cdot 7}e^{12} + \ldots\right)\cos 8M$$

$$+\left(-\frac{3^{12}}{2^{15}\cdot 5\cdot 7}e^9 + \frac{3^{14}\cdot 11}{2^{18}\cdot 5^2\cdot 7}e^{11} - \ldots\right)\cos 9M$$

$$+\left(-\frac{5^7}{2^8\cdot 3^4\cdot 7}e^{10} + \frac{5^8}{2^7\cdot 3^3\cdot 7\cdot 11}e^{12} - \ldots\right)\cos 10M$$

$$+\left(-\frac{11^9}{2^{18}\cdot 3^4\cdot 5^2\cdot 7}e^{11} + \ldots\right)\cos 11M$$

$$+\left(-\frac{2\cdot 3^6}{5^2\cdot 7\cdot 11}e^{12} + \ldots\right)\cos 12M$$

The Logarithm of the Radius Vector.

The logarithm of the radius vector may be expressed in terms of the eccentricity and mean anomaly, by developing the equation $\frac{r}{a} = 1 - e \cos E$ by the logarithmic series, thus,

$$\log \frac{r}{a} = -e \cos E - \frac{e^2}{2} \cos^2 E - \frac{e^3}{3} \cos^3 E \ - \&c.$$

Transforming this series from powers of cos E to the cosines of multiples of E we obtain the following series, which expresses the logarithm of $\frac{r}{a}$ in terms of the eccentricity and the eccentric anomaly.

$$\begin{aligned}
\log \frac{r}{a} = &-\left(\frac{e^2}{2^2} + \frac{3e^4}{2^5} + \frac{5e^6}{2^5 \cdot 3} + \frac{5 \cdot 7e^8}{2^{10}} + \frac{3^2 \cdot 7e^{10}}{2^9 \cdot 5} + \frac{7 \cdot 11e^{12}}{2^{12}} + \ldots\right) \\
&-\left(e + \frac{e^3}{2^2} + \frac{e^5}{2^3} + \frac{5e^7}{2^6} + \frac{7e^9}{2^7} + \frac{3 \cdot 7e^{11}}{2^9} + \ldots\right) \cos E \\
&-\left(\frac{e^2}{2^2} + \frac{e^4}{2^3} + \frac{5e^6}{2^6} + \frac{7e^8}{2^7} + \frac{3 \cdot 7e^{10}}{2^9} + \frac{3 \cdot 11e^{12}}{2^{10}} + \ldots\right) \cos 2E \\
&-\left(\frac{e^3}{2^2 \cdot 3} + \frac{e^5}{2^4} + \frac{3e^7}{2^6} + \frac{7e^9}{2^6 \cdot 3} + \frac{3 \cdot 5e^{11}}{2^9} + \ldots\right) \cos 3E \\
&-\left(\frac{e^4}{2^5} + \frac{e^6}{2^5} + \frac{7e^8}{2^8} + \frac{3e^{10}}{2^7} + \frac{3 \cdot 5 \cdot 11e^{12}}{2^{13}} + \ldots\right) \cos 4E \\
&-\left(\frac{e^5}{2^4 \cdot 5} + \frac{e^7}{2^6} + \frac{e^9}{2^6} + \frac{3 \cdot 5e^{11}}{2^{10}} + \ldots\right) \cos 5E \\
&-\left(\frac{e^6}{2^6 \cdot 3} + \frac{e^8}{2^7} + \frac{3^2e^{10}}{2^{10}} + \frac{5 \cdot 11e^{12}}{2^{11} \cdot 3} + \ldots\right) \cos 6E \\
&-\left(\frac{e^7}{2^6 \cdot 7} + \frac{e^9}{2^8} + \frac{5e^{11}}{2^{10}} + \ldots\right) \cos 7E \\
&-\left(\frac{e^8}{2^{10}} + \frac{e^{10}}{2^9} + \frac{11e^{12}}{2^{12}} + \ldots\right) \cos 8E \\
&-\left(\frac{e^9}{2^8 \cdot 3^2} + \frac{e^{11}}{2^{10}} + \ldots\right) \cos 9E \\
&-\left(\frac{e^{10}}{2^{10} \cdot 5} + \frac{e^{12}}{2^{11}} + \ldots\right) \cos 10E \\
&-\left(\frac{e^{11}}{2^{10} \cdot 11} + \ldots\right) \cos 11E \\
&-\left(\frac{e^{12}}{2^{13} \cdot 3} + \ldots\right) \cos 12E
\end{aligned}$$

Substituting the values of cos E, cos 2E, &c., we have

$$\log\frac{r}{a} = +\frac{e^2}{2^2} + \frac{e^4}{2^5} + \frac{e^6}{2^5\cdot 3} + \frac{5e^8}{2^{10}} + \frac{7e^{10}}{2^9\cdot 5} + \ldots$$

$$+\left(-e + \frac{3e^3}{2^3} + \frac{e^5}{2^6} + \frac{127e^7}{2^{10}\cdot 3^2} + \frac{1741e^9}{2^{14}\cdot 3\cdot 5} + \ldots\right)\cos \mathrm{M}$$

$$+\left(-\frac{3e^2}{2^2} + \frac{11e^4}{2^3\cdot 3} - \frac{3e^6}{2^6} + \frac{3^2e^8}{2^7\cdot 5} + \frac{349e^{10}}{2^9\cdot 3^3\cdot 5} + \ldots\right)\cos 2\,\mathrm{M}$$

$$+\left(-\frac{17e^3}{2^3\cdot 3} + \frac{7\cdot 11e^5}{2^7} - \frac{743e^7}{2^{10}\cdot 5} + \frac{3539e^9}{2^{13}\cdot 3\cdot 5} - \ldots\ldots\right)\cos 3\,\mathrm{M}$$

$$+\left(-\frac{71e^4}{2^5\cdot 3} + \frac{129e^6}{2^5\cdot 5} - \frac{387e^8}{2^8\cdot 5} + \frac{8639e^{10}}{2^7\cdot 3^3\cdot 5\cdot 7} - \ldots\ldots\right)\cos 4\,\mathrm{M}$$

$$+\left(-\frac{523e^5}{2^7\cdot 5} + \frac{10039e^7}{2^{10}\cdot 3^2} - \frac{94739e^9}{2^{13}\cdot 3\cdot 7} + \ldots\ldots\ldots\right)\cos 5\,\mathrm{M}$$

$$+\left(-\frac{899e^6}{2^6\cdot 3\cdot 5} + \frac{6617e^8}{2^7\cdot 5\cdot 7} - \frac{33571e^{10}}{2^{10}\cdot 5\cdot 7} + \ldots\ldots\ldots\right)\cos 6\,\mathrm{M}$$

$$+\left(-\frac{355081e^7}{2^{10}\cdot 3^2\cdot 5\cdot 7} + \frac{986099e^9}{2^{15}\cdot 3\cdot 5} - \ldots\ldots\ldots\ldots\right)\cos 7\,\mathrm{M}$$

$$+\left(-\frac{47259e^8}{2^{10}\cdot 5\cdot 7} + \frac{3959051e^{10}}{2^9\cdot 3^4\cdot 5\cdot 7} - \ldots\ldots\ldots\ldots\right)\cos 8\,\mathrm{M}$$

$$+\left(-\frac{16541017e^9}{2^{15}\cdot 3^2\cdot 5\cdot 7} + \ldots\ldots\ldots\ldots\ldots\right)\cos 9\,\mathrm{M}$$

$$+\left(-\frac{5719087e^{10}}{2^{10}\cdot 3^4\cdot 5\cdot 7} + \ldots\ldots\ldots\ldots\ldots\right)\cos 10\,\mathrm{M}$$

Check. Putting M=0, it reduces to

$$\log \frac{r}{a} = -\left(e + \frac{e^2}{2} + \frac{e^3}{3} + \frac{e^4}{4} + \ \ldots\ + \frac{e^{10}}{10} + \ \ldots\right)$$

as it should.

When the eccentricity is large, as in the case of the orbits of comets and some of the asteroids, the preceding series for the computation of the eccentric anomaly are not sufficiently converging. In that case the following method may be used:

In the equation $M = E - e \sin E$, let $E = M + x$, that is $x = e \sin E$; then we shall have

$$M = M + x - e \sin (M + x)$$

or

$$x = e \sin M \cos x + e \sin x \cos M$$

$$= e \sin M \left(1 - \frac{x^2}{2} + \ \ldots\right) + e \cos M \left(x - \frac{x^3}{6} + \ \ldots\right)$$

or

$$e \sin M = (1 - e \cos M)x + \frac{e \sin M}{2}x^2 + \frac{e \cos M}{6}x^3 - \ \ldots$$

Reverting this series, we have

$$e \sin E = x$$

$$= \frac{e \sin M}{1 - e \cos M} - \tfrac{1}{2}\left(\frac{e \sin M}{1 - e \cos M}\right)^3 + \ \ldots$$

therefore,

$$E = M + e \sin E$$

$$= M + \frac{e \sin M}{1 - e \cos M} - \tfrac{1}{2}\left(\frac{e \sin M}{1 - e \cos M}\right)^3 + \ \ldots$$

It will generally be sufficient to use only the first two terms of this series which will give the first approximate value of E, which we will denote by E′. With this approximate value compute

$$M' = E' - e \sin E';$$

then, by subtraction, we have

$$M - M' = E - E' - e (\sin E - \sin E')$$

$$= E - E' - e (E - E') \cos E', \text{ approximately;}$$

whence

$$E = E' + \frac{M - M'}{1 - e \cos E'},$$

which is a more accurate value of E.

The last process may be repeated as often as necessary, but unless the first approximation is very far from the truth it will not be necessary to proceed with the computation farther than we have indicated.

This is the same as the method recommended by the late Professor Encke in the *Berliner Astronomisches Jahrbuch*, 1838, and is substantially the same as that given by Gauss in his *Theoria Motus Corporum Cœlestium*, art. ii.

Note on Dr. Morrison's Paper. By Professor J. C. Adams, M.A., F.R.S.

The reference to Hansen's paper should be made to *Abhandlungen der Sächsischen Gesellschaft der Wissenschaften*, Band iv. p. 249, instead of to Band ii. as stated by Dr. Morrison.

In this paper Hansen's object is not merely to express the coefficients of the series which gives the eccentric anomaly in powers of e, otherwise this might have been done much more simply in the following manner.

Calling g the mean, and x the eccentric anomaly, we have

$$g = x - e \sin x,$$

or

$$x = g + e \sin x,$$

which is in the proper form for the application of Lagrange's theorem for developing x or any function of x in terms of g and ascending powers of e.

Hence we have

$$x = g + e \sin g + \frac{e^2}{1.2}\frac{d}{dg}(\sin^2 g) + \frac{e^3}{1.2.3}\frac{d^2}{dg^2}(\sin^3 g) + \frac{e^4}{1.2.3.4}\frac{d^3}{dg^3}(\sin^4 g) + \&c.$$

whence by substituting for the powers of sin g their expressions in sines or cosines of multiples of g, and differentiating, we may readily obtain the function of g which multiplies any given power of e.

The numerical coefficient of the term in $(x-g)$ which involves

$$e^m \sin (m-2n)g$$

is

$$(-1)^n \left(\frac{m-2n}{2}\right)^{m-1} \frac{1}{(1.2 \ldots n)(1.2 \ldots \overline{m-n})}$$

where m is a positive integer, and n is either zero or a positive integer less than $\frac{m}{2}$, and $(1.2 \ldots n)$ is to be put $=1$, when $n=0$.

The expressions for x and for the sines of multiples of x are developed to the 12th power of e by Schubert in the appendix to Bode's *Jahrbuch* for 1820. In the same appendix Schubert likewise gives the development of the true anomaly in terms of the mean to the 13th power of e.

Oriani had already given this last-mentioned development to the 11th power of e in the appendix to the Milan "Ephemeris" for 1805.

The numerical coefficients which he finds differ in four cases from those given by Schubert, but I have recomputed the coefficients in these cases, and find that Schubert's results are correct.

There is a misprint, however, in Schubert's expression for the true anomaly at the foot of p. 230, where the coefficient of

$$e^{12} \sin 12g$$

should be

$$\frac{7218065}{2^{13} \cdot 3 \cdot 7 \cdot 11} \text{ instead of } \frac{7218065}{2^{13} \cdot 3^7 \cdot 11}.$$

Delambre's formula is copied from Oriani's, and is therefore affected by the same errors, together with some additional typographical ones.

I have verified Schubert's result for (v), the true anomaly in terms of the mean, by the consideration that when $g=0$, the value of

$$\frac{dv}{dg} \text{ becomes } \frac{(1+e)^2}{(1-e^2)^{\frac{3}{2}}}$$

$$= 1 + 2e + \frac{5}{2}e^2 + 3e^3 + \frac{27}{8}e^4 + \frac{15}{4}e^5 + \frac{65}{16}e^6 + \frac{35}{8}e^7 + \frac{595}{128}e^8 + \frac{315}{64}e^9 + \frac{1323}{256}e^{10} + \frac{693}{128}e^{11} + \frac{5775}{1024}e^{12} + \frac{3003}{512}e^{13} + \text{ &c.}$$

By comparing Schubert's result with that of Dr. Morrison, we see that there are the following errata in the latter: viz. the coefficient of $e^{10} \sin 8\,\mathrm{M}$ in the equation of the centre should be

$$-\frac{4745483}{2^9 \cdot 3^4 \cdot 5 \cdot 7} \text{ instead of } -\frac{1182827}{2^7 \cdot 3^4 \cdot 5 \cdot 7},$$

and the coefficient of $e^{12} \sin 10\,\mathrm{M}$ should be

$$-\frac{76972457}{2^{11} \cdot 3^4 \cdot 7 \cdot 11} \text{ instead of } -\frac{769805651}{2^{12} \cdot 3^4 \cdot 5 \cdot 7 \cdot 11}.$$

In Schubert's expression for $\frac{r}{a}$ in p. 231, which is also carried as far as e^{13}, there are the following errata, which are evidently merely typographical: viz. in the coefficient of $-\cos 3g$, instead of

$$-\frac{3^6 \cdot 11}{2^{17} \cdot 5 \cdot 7} e^{11} \text{ should be } +\frac{3^6 \cdot 11}{2^{17} \cdot 5 \cdot 7} e^{11},$$

and in the coefficient of $-\cos 12g$, instead of

$$\frac{2 \cdot 3}{5^2 \cdot 7 \cdot 11} e^{12} \text{ should be } \frac{2 \cdot 3^6}{5^2 \cdot 7 \cdot 11} e^{12}.$$

Oriani's formula for the radius vector has been examined and found correct.

A very good investigation of the general term of the expansion of the true anomaly in terms of the mean is likewise given in a paper by Mr. Greatheed, in the first volume of the "Cambridge Mathematical Journal," p. 208 (p. 228 in the second edition).

The approximate expression for the eccentric anomaly in terms of the mean given by Dr. Morrison in the latter part of his paper conincides with the first two terms of the series found in Keill's "Astronomical Lectures," p. 291 (5th edition, 1760), and the method of correcting an approximately known value which Dr. Morrison quotes from Encke is identical with Newton's method for the same purpose, which is also explained in Keill's "Lectures," p. 296 *et seq.*

On this subject reference may also be made to my paper in the *Monthly Notices* for December 1882, p. 43.

In addition to the errata already specified, the following may be noticed :—

In Oriani's formula for the equation of the centre, in the Milan "Ephemeris," 1805, pp. 14 and 15,

In the coefficient of sin $4g$,
instead of $-\frac{1367}{2^7 . 3^3 . 7} e^{10}$ read $-\frac{1619}{2^7 . 3^3 . 7} e^{10}$.

In the coefficient of sin $5g$,
instead of $-\frac{3649663}{2^{17} . 3^3 . 7} e^{11}$ read $-\frac{4305913}{2^{17} . 3^3 . 7} e^{11}$.

In the coefficient of sin $6g$,
instead of $+\frac{7751}{2^{10} . 6} e^{10}$ read $+\frac{7751}{2^{10} . 7} e^{10}$.

In the coefficient of sin $11g$,
instead of $\frac{630395121 01}{2^{17} . 3^4 . 5^2 . 7 . 11} e^{11}$ read $\frac{629290171 01}{2^{17} . 3^1 . 5^2 . 7 . 11} e^{11}$.

As Delambre's formula is copied from Oriani, it is affected with the same errors, and in addition to these the following errata occur :—

In the Introduction to Delambre's Solar Tables, 1806,

In the coefficient of sin g,
instead of $\frac{565879}{2^{16} . 3^2 . 5^2} e^{11}$ read $\frac{565879}{2^{16} . 3^3 . 5^2} e^{11}$.

In the coefficient of sin $6g$,
instead of $-\frac{7913}{2^2 . 5 . 7} e^8$ read $-\frac{7913}{2^7 . 5 . 7} e^8$.

In the coefficient of sin $7g$,
instead of $-\frac{1173271}{2^{14} . 3^2 . 5} e^9$ read $-\frac{1773271}{2^{14} . 3^2 . 5} e^9$.

And in his "Astronomy," 1814, vol. ii. p. 52,

In the coefficient of $\sin 2g$,

instead of $+\frac{677}{2^2 \cdot 3^3 \cdot 5} e^{10}$ read $+\frac{677}{2^9 \cdot 3^3 \cdot 5} e^{10}$.

Also in Delambre's expression for $\frac{r}{a}$ the following errata occur:—

In the Introduction to his Solar Tables, 1806,

In the coefficient of $-\cos g$,

instead of $-\frac{3}{2^2} e^3$ read $-\frac{3}{2^3} e^3$.

In the coefficient of $-\cos 5g$,

instead of $+\frac{5^6}{2^{13} \cdot 9} e^9$ read $+\frac{5^6}{2^{13} \cdot 7} e^9$.

And in his "Astronomy," 1814, vol. ii. p. 51,

In the coefficient of $-\cos 5g$,

instead of $\frac{53}{2^7 \cdot 3} e^5$ read $\frac{5^3}{2^7 \cdot 3} e^5$.

Also in Delambre's formula for the hyperbolic logarithm of the radius vector, the following errata occur:—

In the Introduction to his Solar Tables, 1806,

In the coefficient of $-\cos 2g$,

instead of $-\frac{9}{240} e^8$ read $-\frac{9}{640} e^8$.

In the coefficient of $-\cos 8g$,

instead of $\frac{47529}{2^{10} \cdot 5 \cdot 7} e^8$ read $\frac{47259}{2^{10} \cdot 5 \cdot 7} e^8$.

And in his "Astronomy," 1814, vol. ii. p. 50,

In the coefficient of $-\cos 7g$,

instead of $\frac{355081}{2^{10} \cdot 3^2 \cdot 5^7} e^7$ read $\frac{355081}{2^{10} \cdot 3^2 \cdot 5 \cdot 7} e^7$.

On the Orbit of γ Coronæ Australis. By A. M. W. Downing, M.A.

Elements of the orbit of this binary star have been published by Professor Schiaparelli in *Astronomische Nachrichten*, No. 2073, and a comparison of observed and computed places given up to the date 1875·65, from which it appears that these elements represent the series of observations made up to that date extremely well. As several more recent observations of γ *Coronæ Australis* have now been published, I have determined the corrections applicable to Schiaparelli's elements in order that the places computed from them may be brought into rather better accord with the whole series of observations extending from 1834·47 to 1880·67.

The method of computation which I have adopted is that proposed by Professor Klinkerfues (*Theoretische Astronomie*, Vorlesung 110) with Dr. Doberck's modifications, by which corrections to all the assumed elements, except the semi-axis major, are found from six normal position-angles. From a consideration of the outstanding errors of the position-angles computed from Schiaparelli's elements, the following normal places have been found:—

Epoch.	Normal Angle.	θ_c
1835·97	35°·36	35°·46
1851·88	3·52	3·53
1856·34	350·62	350·55
1861·88	327·72	327·56
1876·92	250·39	250·26
1879·56	238·31	238·42

If, now, the mean anomaly at any time t be represented by

$$m + (\mu + \delta\mu)t,$$

where μ is the assumed mean annual motion, by proceeding according to the above-mentioned method the following values are found:—

$$m = +2°·805, \quad \text{and } \delta\mu = +0°·0705;$$

and the corresponding correction to the assumed value of the eccentricity is $-0·0015$. Whence we derive

$$T = 1883·203, \qquad P = 54·985 \text{ years}, \qquad \text{and } e = 0·6974.$$

True anomalies for each normal epoch are found from these values, and from these with the assumed γ, λ and Ω, the corresponding position-angles are computed. And by comparing these position-angles with the normal angles we get six equations of condition for determining corrections to γ, λ and Ω of the form

$$\theta' + \frac{d\theta'}{d\Omega}\cdot\Delta\Omega + \frac{d\theta'}{d\lambda}\cdot\Delta\lambda + \frac{d\theta'}{d\gamma}\cdot\Delta\gamma = \theta;$$

where θ' is the computed and θ the normal position-angle. These equations being solved by the method of least squares give

$$\gamma = 69°\ 17', \qquad \lambda = 283°\ 57', \qquad \text{and } \Omega = 227°\ 23'.$$

The position-angles for the normal epochs found from these elements are given above in the column θ_c. Lastly, the semi-axis major is found from all the observations of distance (combined with the new elements) with the exception of the observation made at the date 1879·70.

The new elements are:—

$T = 1883{\cdot}203$
$P = 54{\cdot}985$ years
$e = 0{\cdot}6974$
$\gamma = 69^\circ\ 17'$
$\lambda = 283\ 57$
$\Omega = 227\ 23$
$a = 2''{\cdot}44$

And the comparison with the individual annual results is as follows:—

Epoch.	Observer.	θ_o	$\theta_o - \theta_c$	ρ_o	$\rho_o - \rho_c$
		°	°	″	″
1834·47	Herschel	37·1	−1·79		
35·55	,,	36·8	+0·40		
36·43	,,	34·5	−0·04		
37·21	,,			2·66	+0·41
37·43	,,	32·7	+0·16		
47·32	Jacob	14·1	+0·03	2·30	+0·10
50·46	,,	5·87	−1·19	2·29	+0·25
51·54	,,	4·47	+0·05	2·26	+0·27
52·49	,,	2·21	+0·29	1·90	−0·02
53·52	,,	359·04	−0·06	1·83	−0·03
53·71	Powell	358·57	+0·02		
54·26	Jacob	356·16	−0·77	1·71	−0·11
54·78	Powell	355·58	+0·22		
55·77	,,	352·93	+0·71		
56·44	Jacob	349·44	−0·53	1·67	−0·03
57·44	,,	347·37	+0·97	1·61	−0·03
58·20	,,	343·42	−0·13	1·53	−0·07
59·72	Powell	338·1	+0·26		
61·69	,,	328·8	+0·32		
62·27	,,	325·3	−0·33		
63·84	,,	318·1	+0·38		
75·65	Schiaparelli	257·41	+0·78	1·45	−0·03
76·65	Howe	253·1	+0·94	1·67	+0·19
77·43	Schiaparelli	248·36	−0·31	1·49	+0·01
77·61	Howe	245·73	−2·11	1·37	−0·10
77·69	O. Stone	249·4	+1·92		
78·49	,,	242·4	−1·35	1·22	−0·22
78·49	Howe	242·9	−0·85	1·47	+0·03
79·70	Burnham	240·0	+2·34	0·87	−0·48
80·46	Russell	233·13	−0·06	1·15	−0·09
1880·67	Hargrave	232·37	+0·54	1·32	+0·12

The corrections to Schiaparelli's elements are quite small, and perhaps of little practical importance. My chief object in bringing the subject forward is to direct attention to the circumstance that, according to these elements the periastron passage occurred in the early part of the present year, in the hope that observers in the southern hemisphere may be induced to make observations of this interesting binary before the present critical state of things has passed away.

Observations of the Companion of Sirius, made at the Dearborn Observatory, Chicago. By Professor G. W. Hough, Director.

(*Communicated by the Secretaries.*)

Date.	Sid. Time.	P.	S.	Date.	Sid. Time.	P.	S.
	h	°	″		h	°	
1880·151		48·7	9·77	1882·145	7·3	42·7	9·32
·173		52·3	9·79	·148	7·0	42·7	9·35
·181		47·8	10·05	·164	7·2	43·0	9·22
				·195	7·3	42·8	9·26
1881·206		45·2	9·47	1882·991	5·2	41·9	8·98
·260		45·7	9·62	1883·066	6·1	39·3	8·81
·269		47·3		·074	6·7	40·8	9·23
·280		44·9		·115	5·5	39·3	9·11
·288		43·3	9·71	·131	6·0	39·5	9·03
				·134	6·1	39·8	8·83
1882·085	6·0	43·8	9·35	·137	5·6	39·2	9·17
·088	6·0	43·8	9·25	·142	7·5	39·5	clouded
·102	6·0	43·1	9·27	·159	6·3	39·7	9·02
·104	7·0	43·4	9·32	·175	6·9	39·6	8·99
·109	6·5	42·9	9·32	·195	7·9	38·0	9·03
			Results.				
1880·168		49·6	9 87	1882·127		43·1	9·30
1881·260		45·3	9·60	1883·120		39·7	9·02

On Hell's alleged Falsification of his Observations of the Transit of Venus in 1769. By Simon Newcomb.

The story of Hell's supposed tampering with his observations of the Transit of *Venus*, made at Wardhus in 1769, is so familiar to all interested in the subject that only a brief mention of its points is necessary. It is, in substance, that he delayed publishing his observations so long as to give rise to the suspicion of intending to alter them; that he showed them to no one until

after he had received the observations made at other stations; that a cloud was thus thrown over their genuineness; that the suspicions thus excited were confirmed in 1835 through the discovery and publication by Littrow of Hell's original MS. journal, which its author had neglected to destroy; and that the examination of this journal showed numerous cases of alteration and erasure of the original observed figures, including the seconds of first interior contact, which had been completely erased and replaced by new numbers inserted with different ink at some subsequent time. And the reason for all this was supposed to be that Hell desired to publish, not his true observations, but results which should be in the best possible accordance with the observations of others.

More precise statements on some points are these:—The Transit occurred 1769, June 3; Hell's party sailed from Wardhus June 27, but meeting with delays from adverse weather, and stopping to make observations, they did not reach Drontheim until August 30. After some stay here and in Christiania, Copenhagen was reached on September 17. The observations were communicated to the Danish Academy of Sciences in November or December; the printing commenced December 13, and on January 13, 1770, Hell received twenty printed copies.* For the statement that Hell was loudly called upon for his observations before he would consent to their publication, I do not know the original authority.

I have taken advantage of a visit to the Imperial Observatory of Vienna to make, with the consent and support of its Director, Professor E. Weiss, an examination of Hell's manuscript. The result is so different from that generally accepted that it seems proper to prepare a statement on the spot, with the documents before me and the circumstances in mind, instead of waiting to make a more elaborate discussion in another place.

The document with which we are principally concerned is a thin MS. volume in folio, containing twenty-seven finely written pages, and nearly as many blank ones, bearing the heading "Observationes Astronomicæ et Cætera in Itinere litterario Viennâ Wardöehusium factæ. 1768. A. M. Hell."

This volume is assumed to be in Hell's own writing, and to be his original journal of his observations. Littrow apparently treats of it as the actual first record of Hell's observations. This seems very improbable. The pages are too large to be held in the hand at the telescope, and the figures of the observations fit too well into the record to suppose that they can be the figures first written. Indeed, it is hardly conceivable that one should have written an account of the circumstances up to the moment of first contact in so continuous a way that the figures of the observation could have been put in at the moment of observation. These figures must therefore have been copied from rough notes

* These dates are from Littrow's work, *P. Hell's Reise nach Wardoe*, Journal of Sajnovics, pp. 143–158.

of some kind made at the telescope, and put in when, after the observation, Hell had an opportunity to write up the journal.

This being admitted, the question might arise whether the journal was certainly written at Wardhus at all, and could not have been written up subsequently from rough notes. Circumstances too numerous to be detailed show that the writing was done at the station, probably at the close of each day's work or each set of observations.

In connection with this journal we have to consider Littrow's book: *P. Hell's Reise nach Wardoe bei Lapland und seine Beobachtungen des Venus-Durchganges im Jahre* 1769, *von Carl Ludwig Littrow.* (Wien, 1835.) This book contains extracts from the MS. journal above mentioned, accompanied by a copious commentary descriptive of the writing, and pointing out the alterations in the record.

Hell's own publication, of which the first edition appeared at Copenhagen in 1770, should also be compared.

A comparison of these writings in a cursory way brought out one circumstance which has always been overlooked. What Hell sent to press in December 1769 was not a transcript of this journal, but a more copious account, containing eighty-one printed pages, with only an occasional identity of language. But, with a single unimportant exception, the numbers are all printed without change from the original MS. journal, whether corrected or uncorrected in that journal. Now, this fact can be reconciled with the supposition that the printed numbers were the result of calculation from the observations of others in only a single way: the journal must have been corrected from the printed book; for it is contrary to human nature to suppose that if Hell had once set out to publish numbers derived from extraneous sources he could have failed to find occasion to correct them from time to time, as new data arrived and new computations were made, and the final numbers could have been reached only when the book was printed.

Now, it is very clear to me, and the evidence will be given presently, that nearly all the alterations were made at the station—two, at least, before the ink got dry. I therefore conclude from this general survey that *whatever the sources from which the corrections were derived, the numbers as printed by Hell were all but one or two obtained at Wardhus.*

I now go into the corrections more in detail, and begin with two bad-looking cases, having, however, no direct reference to the Transit.

I. The times of the corresponding altitudes observed on June 4 are so corrected and altered as to bring about a better agreement among the individual results. Measured by our standard, this was dishonest, but, as I have elsewhere shown, more than one astronomer of good repute in the last century considered it quite right and proper. Considering the matter in a practical and not in a moral light, I submit without discussion

the proposition that, from the fact that an observer is capable of altering the individual observations of a series made by himself to make them consistent, it does not follow that he would alter his own observations to bring them into agreement with the results of a long and laborious calculation from the not more certain observations of others.

II. Having observed the last of the above-mentioned altitudes at $10^h\ 8^m\ 18^s$ (a record which he leaves unaltered), he added, originally :—

"N.B. ante hanc positionem minuto 8 10″ mane observatum est initium eclipsis ⊙, quod mihi jam 20 secundis circiter citius factum fuisse videtur." After his return to Copenhagen he altered the 10″ to 27″, and the 20 to 6, which makes the observation *post* instead of "ante." The change of wording *ante* to *post* is made in the printed book, but not in the journal.

Now, since Hell made no attempt at a serious observation of the beginning of the eclipse, but was (it is evident) content with incidentally remarking it as he was taking time observations with his quadrant; since, also, it came within 10^s of an observation of altitude, so that he could not independently have noted it, he might well have been uncertain as to whether it was 8^s or 9^s before or 8^s or 9^s after the altitude time, and have changed his opinion after writing the note. Indeed, in the printed book he speaks of the numbers as a mere guess : "accipiebam correspondentes." Again, looking at the matter from a practical standpoint, this very alteration may supply us with a ground for believing that Hell's corrections were not made with the object of deceiving, for he could have had no such object in this alteration. He lays no stress on the observation, expressly says that such a contact cannot be accurately observed, and does not use it to determine his longitude.* Only the observed end of the eclipse he considered available for this purpose, and *he gives this without alteration.*

* Since writing this sentence, I find its last statement curiously at fault. It is true that Hell repeatedly expresses the view here attributed to him, remarking : "Et cum verus in Initio Eclipsium Solarium contactus limbi Lunæ cum limbo Solis sit observatu impossibilis . . . solo utar fine Eclipseos variis in locis observato" ; and he carries through his discussion on this principle. But when he comes to the Wardhus observations, he suddenly brings in an observed time of beginning, $21^h\ 22^m\ 47^s$, and actually pretends to give it the same weight as the observation of end. Between this time and the altered record in the journal there is a break of continuity arising from the fact that neither in MS. nor print does he give the clock error he applied. But the clock error computed from his data is $45^m\ 32^s$, which would give $10^h\ 8^m\ 19^s$ as the adopted clock time, which corresponds to nothing, but is nearest to the altitude itself. It might be supposed that the estimated 5 or 6 seconds was subtracted before applying the clock error, but in one place he expressly says this is still to be done. The substance of the case is this:—The observed end he gives accurately and faithfully, and uses it correctly. The beginning he frankly acknowledges to have been nothing but a series of guesses, yet, having adjusted the guesses to give a longitude only 2^s different from that of the end, he treats it as a sound result.

We now go back to the observations of contact, beginning with first contact. In his journal he first wrote: "Venus hora 9. m. 1?′ 35″ observata est, jam parte aliqua sui diametri ingressa, ita ut existimem primum contactum 20″ circiter ante factum." The figure in the place (?) is illegible in consequence of having been blotted out with the finger before the ink got dry. There is a vague suggestion of a figure 4 in the blot, but superposed on this a much better defined loop, apparently the top of a figure 6, as Hell used to write it. But the bottom of the 6 is totally wanting, though it ought plainly to show. Then over the blotted figure is written a heavy figure 5, and over the 35 is written an equally heavy 17, thus making the time $9^h\ 15^m\ 17^s$. An attempt was also made to blot out the 2 in the 20 by a stroke of the finger before the ink got dry, but enough of the figure was left for identification. Over it is written 3, making the estimate 30^s instead of 20^s.

It is quite clear to me that the alterations were made with the same ink as the original, apart from the testimony afforded by the blots; for, although the corrections look strikingly darker than the rest of the writing, we find that wherever an unusually thick layer of ink happened to run from the pen, it looks almost as dark.

On this alteration Littrow remarks that it was probably made "nach der erst später, aus dem Austritte bekannt gewordenen Verweilung des Venusdurchmessers am Sonnenrande." I cannot explain this suspicion. The "Verweilung aus dem Austritte" was $18^m\ 9^s$, which, subtracted from the time of internal contact at ingress, gives $9^h\ 14^m\ 32^s$ for first external contact. This is 45^s earlier than the corrected time, and 15^s earlier after the estimated 30^s is subtracted.

For more light we refer to the printed account, which is very explicit. The three men were at the telescopes, an assistant at the clock, carefully watching the face. Whoever saw a small notch in the expected place "illico exclamaret," while "famulus" was to note hour, minute and second by the clock. Borgrewing called first, and immediately afterwards Sajnovics; then Hell, looking into his telescope, saw it. "Erat autem momentum, quo D. Borgrewing & P. Sajnovics exclamarunt, famulo indicante in horologio Viennensi $9^h\ 15'\ 17''$."

Returning to the MS., it looks to me as if a 6 was first written; before the ink was fully dry a 4 was written over it, but immediately erased with the finger and changed to a 5. However this may be, I see no more plausible explanation of the change than some accidental error in first writing down the time in the journal after the observation was over. It is not stated whether "famulus" communicated his time by speech or writing, and I cannot but suspect that the entire minutes were partly guess-work. But the coincidence of the number of seconds (35) with that given in the next following record, that of internal contact, is at least suggestive. Of course Hell had his rough

record of the contact before him when he wrote. Can it be supposed that, in the hurry of writing he took the seconds out of the wrong line, but not the minutes? If so, it would not only explain the error, but would afford positive proof that the disputed record of internal contact was before his eyes at Wardhus when he wrote.

I can only regard one conclusion as certain: that the corrections were made at the time of writing, and without the slightest intention of giving anything but the actually observed moment when *Venus* was first seen.

We now come to the much-disputed observations of internal contact, which appear in the journal in the following form:—

	h	m	s	
"Videtur contactus fieri	9	32	35	
Contactus certus visus		32	41	
fulmen		32	48	
Pater Sajnovics suo tubo contactus dubius	9	32	30	30
certissimus ut ajebat ...		32	45	45"

At first sight the figures of seconds, 35 and 41, in Hell's observations seem to be corrected figures. Littrow remarks upon them that the paper here and below bears marks of having been scraped, and that the original figures of seconds had been carefully erased; in consequence the ink had spread in the paper. But one sees at a glance that the latter statement is erroneous. The figures have not that diffused outline which arises from the absorption of ink in the tissue of the paper, but are, on the contrary, well defined with perfectly sharp edges and no loss or change of tint near the border. It is, however, true that when one looks carefully he receives the impression that the paper has been scraped all the way down the column of seconds, and especially where the observations of Sajnovics are written. But we may also note that this is near where the paper has been folded, and that there are diversities of the same sort here and there in other parts of the MS., which evidently are the mere effect of age and accident. Moreover, it is only Hell's figures of seconds which show the slightest trace of being written on a scraped surface.

Happily the question of erasure admitted of an easy and decisive settlement. The paper is of a kind which I believe is now called "wove," and of which the original ribbed surface has not been smoothed off by glazing. It was therefore examined in a dark room by direct sunlight shining through an opening, and the paper was held so that the rays should strike it at a very small angle. The ribbed texture of the surface was then brought out in bold relief of light and shadow, and it was clearly seen that the ribs extended continuously through Hell's disputed figures without the slightest break or loss of strength, except one little depression between the lines where it looked as if a

dot had been dug out with the point of a knife. Over the observations of Sajnovics the ribs are not so distinct, but they were most obliterated in the column of minutes, about which there is no dispute. Moreover, these variations are seen everywhere on the paper.

To put the matter still further beyond doubt, I asked Director Weiss's permission to write on one of the blank pages and try whether it were possible to erase the figures without injuring the texture of the surface. It was not found possible when the test of oblique sunlight was applied.

The question of erasure thus disposed of, our attention is once more directed to Hell's heavy figures, 35 and 41. We first remark that Hell's style of figure is so distinctive that in all other cases where he has written one figure over a different one, no matter how heavily, parts of the original figure can be seen protruding, and, when not blotted out with the finger, can be identified. But in this case, there is but a single trace of a figure under those written—namely, when examined with a magnifying glass, we see under one edge of the 4 what seems to be part of an original 4.

The evidence is therefore conclusive that no different figures from those now visible were ever written here. If, then, they are in any way the result of calculation from other observations, the place must have been left blank until Hell got back to Copenhagen, and made the necessary calculations. This hypothesis seems to me too fanciful for serious discussion, especially as we have an obvious explanation of the suspicious character of the figures. The ink did not always flow well from the pen, and figures as well as letters are frequently retouched in this part of the MS. Noticing that the figures as he first wrote them were not so complete and distinct as he desired them, he immediately wrote them over again in the spirit and with the hand of a man who was determined that no imperfections of pen and ink should be allowed to render so important a record doubtful.

This explanation is possibly in contradiction with a statement of Littrow that the figures 35 and 41 are made in much darker ink than that of the original. I say "possibly," because I am unable to infer from Littrow's statement exactly what individual figures and words he meant to say were written with this "viel schwarzerer Tinte." But that he was entirely at fault in saying that the corrections and alterations here described were made in different ink, my eyes do not permit me to doubt. The ink, of course, looks darker where the figures are heavily re-written, but this is only because the layer of ink is thicker. The tint is obviously the same.

We now reach a part of the record which looks more suspicious. The line "fulmen 9 32 48" is not only an interlineation, but is written in decidedly different ink from all the original MS.—an ink which has not faded so much as the other, and so has almost a blue tinge by contrast. That Littrow, in

arraying his proofs of Hell's forgery, should have failed to dwell upon the obvious difference between this ink and that with which the alterations were made leads me to suspect a defect in his sense of colour.* The original journal, up to the time that Hell left Wardhus, being all written in one kind of ink, we conclude that the insertion was made after he reached Copenhagen. To judge of its meaning and origin, we must refer to Hell's printed book, where the time is given as that of the formation of the thread of light. Moreover, this time, and that of certain contact, have reference to foot-notes in which it is remarked that some observers take the one, and some the other, for the true contact. I therefore conclude that he did not add this phase until he had seen the observations of others. We therefore meet the question, Was it a manufactured time, the material for which was furnished by other observations, or was it fixed solely from what he saw at Wardhus? Two circumstances seem to me to render the first hypothesis very improbable.

The first is, that the time $32^m\ 48^s$ is uncorrected in the MS. and identical with that in the printed book. A time concluded from other sources would have been subject to so many alterations as new data came in, that we could not expect this coincidence.

The other is, the shortness of the interval, 7^s, between the two phases. The observations in Western Europe, which Hell must have first seen, were made with a very low altitude of the Sun, and so gave differences ranging from 10^s or 20^s up to 60^s. A difference manufactured from these data could not have been so small as 7^s.

Two hypotheses are before us as to how the insertion was determined. We may suppose that Hell, when he found he had omitted what other observers considered an important phase, tried to remember how long after the recorded contact he first saw the Sun's limb continuous, and wrote the result in his journal; or we may suppose that he made a memorandum at the time of the observation, but omitted to copy it in the journal, either through inadvertence, or because he deemed it too late for contact. When he found the phase important, he merely copied the omitted record in his journal.

The use of the queer word "fulmen," which appears only in the MS., seems to me to give colour to the last hypothesis. We can hardly conceive of one using it deliberately, after six months, to express the formation of the thread of light; whereas at the

* At the time of writing this sentence (which he has been careful not to alter) the writer had no idea that any defect was known to exist in Littrow's sense of colour. But on reviewing the comparison of his description with the MS., the case seemed even stronger than the writer had put it. He therefore made inquiries of an authority likely to be well-informed, whether Littrow had ever been suspected of colour-blindness, and was informed in reply that he really was colour-blind to red, and could see no difference between the colour of *Antares* or *Aldebaran* and that of other stars. After receiving this information, no further note was made of Littrow's views of the different inks used.

moment of observation, in the excitement and hurry, it would be a very natural single word to designate the rapid increase of the effulgence of solar light around the following limb of *Venus*, which follows true contact at ingress.*

In printing, the 41^s of the MS. has, intentionally or through inadvertence, been changed to 42^s. I have no explanation of this to offer. Nor can I explain the double column of seconds in the record of Sajnovics' times.

We come now to egress. The times of Hell's notes of the "gutta nigra" are each increased by 2^s, but obviously this correction was made at the time of writing. More serious is a correction of Sajnovics' time. As originally written, they read—

"Pater Sajnovics contactus dubius ... 15 26 20
certus ... 26"

So far as can be inferred from the MS., the first second might have been 26 as well as 20, only then the two times would have been the same, which is improbable. The last line, "certus . . . 26," and the word "dubius," were then struck out on the spot, and the word *certus* substituted for *dubius*. Whether this was merely a suppression of the "contactus dubius," or included also a change of the "contactus certus" from 26 to 20, we cannot say, the MS. being torn where the top of the 6 belongs; but the latter seems more probable, as otherwise there would have been no object in the change. But this is not all. The 20 or 26 is again changed to 18, and so printed. Moreover, this last change appears to be made with a different ink, and, so far as can be judged from so small a surface, the same ink with which the line "fulmen" was written. The explanation is too obvious to need more than a statement. An observation of contact is not like one of a star transit, in which the observer must observe a moment which he cannot alter. It can be only an estimated mean moment for a gradually changing phenomenon extending through a number of seconds. This estimate is liable to change in the mind of the observer as he subsequently thinks over the progress of the phenomenon as he saw it. I should be inclined to accept a change of opinion thus reached, if it were not suggested by a comparison with the results of others. Now, Sajnovics was the constant companion of Hell, both on the journey and while the observations were going through the press. They, no doubt, discussed their times, and in consequence of such discussion Sajnovics concluded that his times were late.

In the exterior contacts the only correction is a change of Hell's time of "contactus dubius" from 20^s to 22^s, while "certus" remains 26^s. I attach no importance to this change, which was evidently made at the time of writing.

* I regard it as a strong confirmation of this view that Mr. Stone, without apparently having made any comparison with Hell's printed observations, reached this same conclusion as to the probable use of the word "fulmen." See *Monthly Notices* R. A. S., vol. xxix. p. 242.

To make this discussion more complete, we may now consider certain collateral circumstances which have been supposed to cast suspicion upon Hell's intentions. First and most oft repeated among these is a supposed delay in making known his observations. The actual facts have already been mentioned. It may be added that, on his return to Copenhagen, Hell proceeded to prepare a complete account of his instruments, observations, and results, which included an investigation of his quadrant and clocks, a discussion of his latitude, longitude, and time, and a full statement of his observations. The whole paper, extending to 80 closely-printed pages, was written, printed and ready for distribution four months after his return to Copenhagen. Not only do I see no suspicious delay here, but it seems to me difficult to suppose that he could have had time to make so complete a reduction of the observations of others as to be able to compare them with his own.

It is true that he did not publish his observed times of contact in advance, as many others did. The reasons he is said to have assigned for this course—such as the amount of computation necessary, and the command of the King not to publish in advance of the official paper—are declared by Encke unworthy of the slightest consideration. They seem to me quite sound reasons. When the party got back to Copenhagen they had an audience of the King, and asked his consent to dedicate their observations to him. That observations made under his auspices, dedicated to him, and published by his Academy of Sciences, should first appear in official form, seems to me a very proper feeling, especially when we consider that, owing to the position of the station being unknown, publication in advance could have served no useful purpose.

Littrow also notes as suspicious circumstances certain expressions of devout sentiment which are scattered here and there in Hell's journal, and which he seems to regard as efforts to compound with his conscience for the sense of guilt with which it was oppressing him. This is possible. But it is equally possible that these expressions were the result of a generally devout state of mind not at all incompatible with moral integrity. Extraneous circumstances will decide the question. Each volume of Hell's *Ephemerides Vindobonienses*, so far as I have noticed, concludes with a similar sentiment. A curious incident mentioned by Sajnovics may be added. The extraordinary continuance of adverse weather which the ship encountered on her return voyage led Hell to investigate the morality of her navigators. Finding that they had on board certain contraband fish, he saw in this circumstance an explanation of the weather, and tried to get this part of the cargo disposed of. Failing in his effort, he left the ship in port and took another, after which there was an immediate cessation of the contrary winds.

In the preceding discussion I have made but slight allusion to the absence of many circumstances which we might expect to

accompany manufactured observations. I have, however, endeavoured to present all the positive evidence within reach so fully as to enable everyone to draw his own independent conclusions. My own conclusions are:—

First. The belief that there was any suspicious delay in the publication of Hell's observations, or anything in his course to give reasonable ground for a suspicion that he intended to tamper with his observations is a pure myth.

Secondly. Excepting the time of formation of the thread of light at ingress, excepting also a discrepancy of one second in the time of internal contact, and a change of two seconds in one of Sajnovics' times, it is proved, not only negatively and presumptively, but by positive evidence and beyond serious doubt, that all the essential numbers of observation given by Hell, whether relating to the transit, time, or longitude, are printed as concluded upon and written in his journal at Wardhus, before there was any possibility of communication with other observers.

Thirdly. The addition of the time of the formation of the thread of light was suggested by the accounts of other observers, but the time itself is Hell's own, obtained possibly from estimation and memory, but more probably from a memorandum made at the time of observation which he neglected to insert in his journal.

Fourthly. The alterations in Sajnovics' time of second internal contact were probably made because Sajnovics himself afterward concluded that his recorded time was too late; but it may be assumed that in reaching this conclusion he was influenced by Hell's observations.

A few words may be added respecting the writer's own proceedings in investigating this subject. In commencing the examination of Hell's journal he had no hope of doing more than decide whether it was or was not safe to use Hell's numbers as actual results of observations, and no thought of doubting the commonly received view of the case. A few notes respecting the different kinds of ink and writing were made as memoranda. Afterwards these notes were compared with Littrow's description of the MS., and the writer was perplexed to find himself differing entirely from the conclusions of Littrow. Not till the paper was nearly written did he make the inquiries which elicited the fact that Littrow was colour-blind. In justice to the latter a further explanation of the source of his probably incorrect judgment should be considered. Before he found the MS., suspicion had rested upon Hell's truthfulness. When he looked into the MS., and saw such extensive alterations in an ink seemingly different from that of the journal, the indictment seemed so clearly proved that his only duty was to make the facts which proved it known to the world. He thus unconsciously assumed the tone of a public prosecutor, and saw all the circumstances from an accuser's point of view.

The Nucleus of the Great Comet (b) 1882.

By A. Ainslie Common.

When this comet was first seen on the morning of Sept. 17, 1882, before perihelion, the nucleus appeared, when viewed with a 6-inch Helioscope, to be large, bright, and quite round, with a diameter of about 45″; although this is only an estimation by comparison with a spot then seen on the Sun, it is possibly not very much in error. The tail was then very bright, and had the appearance shown in the sketch No. 1, which is a copy of the sketch made at the time of the observation, Sept. 16, 23^h to 24^h. Clouds prevented the further observation of the comet, and it was not seen again until Oct. 3, and then only for a short time.

On the morning of Oct. 30 the comet came within reach of the 3-foot reflector, when the nucleus had the appearance of a line of light about 58″ in length; no particular condensation of this line was then noticed, but the morning was not very good, and the time of observation cut short by clouds. On the following morning this line of light was seen clearly divided in its length by a gap in the middle; the edges of this gap were much brighter than the other parts, and formed, as it were, two nuclei, the following one being the brightest. At 17^h 36^m (Oct. 30, 1882), the distance between the centres of these nuclei was 11″. At 17^h 41^m the position angle of the line of light forming the head of comet was 115° 5′, and its width about 10″.

The sketch No. 2 shows the appearance on this morning.

On Nov. 1, at 17^h 2^m, the appearance of the comet was, with a low power, like the great nebula in *Andromeda*, excepting that the centre was condensed to a line of light; with higher powers, the comet appeared as seen the morning before. At 17^h 18^m, the position-angle of head was 117° 5′, and the breadth of the head or line of light 11″, the length being 100″.

On Nov. 2, at 17^h 2^m, a bending of the head was noticed concave to the north, and a slight brightening of the *following* end of the head having the appearance of another nucleus.

Subsequent observations on Nov. 5, 8, 9, 10, and 17 did not show any remarkable change, but on these days eye-observations were confined to hasty looks in the intervals of photographic experiments.

On Jan. 27, 1883, when the comet was visible in the evening, observations between 8^h 52^m and 9^h 25^m showed a remarkable change in the head of the comet; with a low power on a small telescope, and also with the 3-foot, the appearance was that of a long, brightish nebula, no tail being easily traced. With high power on the large telescope, five points of light became visible in the head, and a slight extension on the following side showing the tail. These points of light were in a straight line, and sur-

S

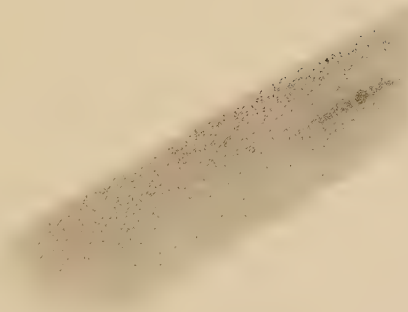
1

1882, Sept. 16th

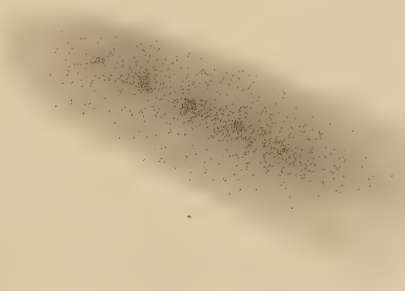
2

1882, Oct. 30th

P



F



3

1883, Jan. 27th

N

W.H. Wesley lith.

The great Comet (b) 1882.
A.A. Common.

West, Newman & Co imp

rounded with a faint haze, fading away into the sky, the south side being the brightest or most easily seen.

Taking the sketch 3, and starting from the preceding side, and calling the five points *a*, *b*, *c*, *d*, *e*, the order of brightness was: *b* and *c* equal, and equal to 11 mag. stars; *a* and *d* about one third as bright, and *e* faintest. The distance between *b* and *c* was 31″·5. The distance between *a* and *e* was 135″·5. The position-angle of the line formed by the 5 points was 240° 20′.

On Jan. 30 a short view of the comet was obtained, when the appearance was unaltered. Position-angle of *b* and *c*, 245° 5′.

On Feb. 5 the comet was well seen about 80″ north of an 11 mag. double (position 256° 50′, distance 5″). The point *c* was the brightest, otherwise the relation was the same.

The position-angle of the two points was, at 8^h 56^m, 246° 15′.

On Feb. 24 the comet was well seen, 8^h 15^m to 8^h 35^m. The position-angle of the two brighter points, *b* and *c*, was 256° 22′ (mean of 4 measures).

The line of points or nuclei was noticed as concave to the N.; the order of brightness was as on last observation—*c* brightest, then *b*, *d*, *a* and *e*; the distance from *b* to *c*, a mean of 3 measures, gave 33″·13.

From Jan. 27 to Feb. 24 the general appearance was much the same. On the first-named day a faint star was noticed *p*. the first nucleus in such a position that it might be taken for another nucleus symmetrically placed with regard to the others, but the motion of the comet proved it to be a star.

On Feb. 24 I strongly suspected another nucleus following the last in order, and in such a position that it would be symmetrically placed. This, however, may have been a star. The comet was bright enough to be well seen, and it is only owing to the limit of my view that further observations have not been made.

1883, *May*.

Post-Perihelion Observations of the Great Comet (b) 1882.

By John Tebbutt.

The only accurate observations I was enabled to obtain of the Great Comet previously to its perihelion passage are those recorded in the *Monthly Notices* for November last. I have several absolute extra-meridian determinations of position on the day before perihelion, and on subsequent days, but as these can be depended on only to the nearest minute of arc they are scarcely worth publication. Of the post-perihelion positions now sent the first four depend on the 3-inch Transit Instrument. As bright stars were observed in sunlight, when possible, in combination with the comet, the deduced Right Ascensions on the

meridian will doubtless be found to be pretty accurate. The North Polar Distances must, however, be regarded as only approximate, seeing they are derived from the readings of a setting-circle whose verniers subdivide to 20″, and by estimation to 10″. All the positions after Sept. 21 were obtained with the $4\frac{1}{2}$-inch Equatorial, the filar micrometer being employed on Sept. 28, Oct. 10 and 12, and in the second set of comparisons for Oct. 7. All the other comparisons were made with an excellent square-bar micrometer. The changes which the comet's nucleus underwent after the perihelion rendered it difficult to decide as to what part of it should be observed. Towards the close of September the nucleus was large and circular, but on the 30th of that month it was first noticed to be elongated in the direction of a parallel of declination. On Oct. 2 its major axis was estimated to be three or four times its minor, and throughout the month the nucleus gradually increased in length, but not correspondingly in breadth. The inclination of the major axis to the parallel of declination gradually increased till about Nov. 22, when it was 45°; the nucleus, however, had become comparatively broader, and was very diffused and ill-defined. The second position for Oct. 7 and the positions for Oct. 10, 13, 14, and 16, refer to a small point of condensation situated about one-fourth of the major axis from the *following* or eastern extremity; all the other positions are those of the centre of the nucleus. During the latter part of October and early part of November the nucleus was more condensed at its extremities than in the middle, the *following* extremity being much the brighter. At the close of November this characteristic had disappeared. On Oct. 26 I adopted the plan of observing the contacts of each bright extremity of the nucleus with the four edges of the bar-micrometer, and thus obtained data for determining the coordinates of the centre and of the extremities. In the columns headed $\Delta\,\alpha$, $\Delta\,\pi$, I have given the corrections, always positive, to be applied to the tabulated R.A. and N.P.D. of the centre in order to obtain the corresponding coordinates of the brighter or *following* extremity. These corrections applied with the contrary sign will of course furnish the coordinates of the *preceding* extremity. During the latter half of November it was impossible to observe the ends of the nucleus with precision. The expediency of the above-described method of observation is fully illustrated by the fact that at Cordoba (see *Ast. Nach.* No. 2481) the *preceding* extremity of the nucleus was observed, though it is rather remarkable that it was recorded as the brighter.

During December and January the comet appeared as an ill-defined and somewhat oval nebulosity, slightly condensed towards the centre, but during February its oval form was not so well marked. During these months the centre only was observed. The comet is now barely visible in the telescope.

Windsor Mean Time.	Comet's Apparent R.A.	$\Delta\alpha$	Comet's Apparent N.P.D.	$\Delta\pi$	Parallax Factors for R.A. Log. $\frac{p}{P}$	Parallax Factors for N.P.D. Log. $\frac{q}{P}$	No. of Comp.	Comp. Star.
d h m s	h m s		° ′ ″					
1882, Sept. 18 23 34 7	11 25 32·72		89 10 35·3			+9·7499		
19 23 22 47	11 18 7·75		90 3 25·1			+9·7401		
20 23 13 4	11 12 19·46		90 51 32·7			+9·7308		
21 23 4 19	11 7 29·86		91 34 46·6			+9·7219		
28 17 30 18	10 46 33 54		95 39 12·2		−8·7242	+9·7174	1	1
29 17 33 51	10 44 21·04		96 10 13·5		−8·7181	+9·7121	1	2
30 17 7 29	10 42 22·03		96 40 8·2		−8·7313	+9·7164	2	2
Oct. 2 16 58 44	∗ + 2 36·99		∗ + 12 22·0		−8·7307	+9·7110	5	3
2 17 2 55	∗ + 4 58·91		∗ + 17 49·5		8 7283	+9·7094	2	4
3 17 0 51	10 36 50·67		98 7 27·1		−8·7266	+9·7057	7	5
5 16 29 30	10 33 35·59		99 2 30·7		−8·7385	+9·7103	4	6
5 16 29 30	10 33 34·96		99 2 29·1		−8·7385	+9·7103	4	7
5 16 36 54	10 33 34·48		∗ − 2 46·6		−8·7350	+9·7070	6	8
6 16 34 28	∗ − 1 2 64		∗ − 2 25·5		−8·7339	+9·7032	3	9
6 16 41 56	10 32 1·84		99 29 59·7		−8·7296	+9·6997	6	10
7 16 17 56	10 30 32·20		99 56 12·0		−8·7400	+9·7072	5	11
7 16 56 41	10 30 30·13		99 56 56·4		−8·7159	+9·6868	11	11
10 16 10 29	10 26 14·43		101 14 19·7		−8·7376	+9·6963	9	12
12 15 46 42	10 23 30·34		102 4 32·0		−8·7450	+9·7007	2	13

Windsor Mean Time.	Comet's Apparent R.A.	Δα	Comet's Apparent N.P.D.	Δπ	Parallax Factors for R.A. Log. $\frac{p}{P}$	Parallax Factors for N.P.D. Log. $\frac{q}{P}$	No. of Comp.	Comp. Star.
d h m s	h m s		° ′ ″					
1882, Oct. 13 16 44 54	10 22 7·40		102 30 28·7		−8·7016	+9·6560	2	14
14 15 57 56	10 20 49·62		102 54 25·2		−8·7355	+9·6816	7	15
16 15 46 37	∗ + 12 18·30		∗ + 1 15·4		−8·7375	+9·6775	6	16
19 15 14 51	10 14 13·12		104 54 0·8		−8·7481	+9·6838	3	17
19 15 23 18	10 14 12·93		104 54 4·5		−8·7440	+9·6768	4	18
19 15 53 4	10 14 12·32		104 55 10·5		−8·7244	+9·6520	13	19
22 15 22 54	10 10 10·16		106 4 28·1		−8·7373	+9·6558	5	20
22 15 22 54	10 10 10·09		106 4 32·5		−8·7373	+9·6558	5	21
23 15 46 26	10 8 46·56		106 28 3·4		−8·7151	+9·6259	7	20
23 15 46 26	10 8 46·33		106 28 4·6		−8·7151	+9·6259	7	21
26 15 39 19	10 4 32·57	2·75	107 36 17·0	0·7	−8·7083	+9·6058	5	22
26 15 48 40	10 4 32·12		107 36 27·8		−8·6986	+9·5962	4	22
30 15 1 53	∗ + 2 49·97	2·34	∗ − 1 38·1	5·7	−8·7277	+9·6089	11	23
30 15 1 53	9 58 38·53	2·34	109 4 56·4	5·7	−8·7277	+9·6089	11	24
30 15 1 53	9 58 38·74	2·34	109 4 46·5	5·7	−8·7277	+9·6089	11	25
Nov. 3 15 51 28	9 52 11·10	3·01	110 32 29·7	7·6	−8·6397	+9·5013	3	26
6 15 8 43	9 47 5·16	2·84	111 35 35·3	14·7	−8·6833	+9·5191	7	27
6 15 8 43	9 47 5·15	2·84	111 35 30·6	14·7	−8·6833	+9·5191	7	28
6 16 1 11	9 47 0·78	2·71	111 36 14·6	9·9	−8·5901	+9·4457	6	29

Windsor Mean Time.	Comet's Apparent R.A.	$\Delta\alpha$	Comet's Apparent N.P.D.	$\Delta\pi$	Parallax Factors for R.A. Log. $\frac{p}{P}$	Parallax Factors for N.P.D. Log. $\frac{q}{P}$	No. of Comp.	Comp. Star.
d h m s	h m s		° ′ ″					
1882, Nov. 7 16 6 45	9 45 13·33	3·02	111 57 15·8	13·5	−8·5648	+9·4222	4	28
7 16 6 45	9 45 13·29	3·02	111 57 10·7	13·5	−8·5648	+9·4222	4	29
9 15 13 35	9 41 34·70	3·00	112 37 26·0	12·9	−8·6523	+9·4692	7	30
9 15 13 35	* + 4 5·01	3·00	112 37 26·0	12·9	−8·6523	+9·4692	7	31
9 15 13 35	9 41 35·04	3·00	112 37 26·3	12·9	−8·6523	+9·4692	7	32
10 14 47 41	9 39 42·36	2·98	112 57 15·4	15·4	−8·6846	+9·4945	10	33
10 15 46 17	9 39 37·01	3·09	112 58 3·5	14·9	−8·5754	+9·4016	5	33
14 13 54 56	9 31 40·51	4·25	114 14 33·8	19·3	−8·7249	+9·5230	10	34
14 13 54 56	9 31 40·57	4·25	114 14 56·8	19·3	−8·7249	+9·5230	10	31
18 14 36 21	9 22 47·35	3·70	115 29 42·5	25·9	−8·6312	+9·3779	6	36
20 15 30 53	9 17 59·60	3·31	116 5 11·8	24·9	−8·4508	+9·2251	7	37
20 15 30 53	9 17 59·62	3·31	116 5 14·5	24·9	−8·4508	+9·2251	7	38
20 15 30 53	9 17 59·51	3·31	116 5 14·8	24·9	−8·4508	+9·2251	7	39
20 15 36 15	* − 5 30·25	3·27	* +15 8·3	23·8	−8·4303	+9·2158	6	40
22 14 30 14	9 13 12·74	3·03	116 37 55·5	35·9	−8·5944	+9·3059	10	41
22 14 30 14	9 13 12·41	3·03	116 37 51·6	35·9	−8·5944	+9·3059	10	42
30 15 19 40	8 51 44·31	2·69	118 34 39·1	53·8	−8·1711	+8·9793	1	43
30 15 19 40	8 51 43·48	2·69	118 34 38·8	53·8	−8·1711	+8·9793	1	44
Dec. 1 12 48 32	8 49 8·92		118 45 17·2		−8·6850	+9·3495	5	45

Windsor Mean Time.	Comet's Apparent R.A.	$\Delta\alpha$	Comet's Apparent N.P.D.	$\Delta\pi$	Parallax Factors for R.A. $\log\frac{p}{P}$	Parallax Factors for N.P.D. $\log\frac{q}{P}$	No. of Comp.	Comp. Star.
d h m s	h m s		° ′ ″					
1882, Dec. 1 12 48 32	8 49 8·49		118 45 14·8		−8·6850	+9·3495	5	46
1 12 48 32	8 49 8·69		118 45 13·7		−8·6850	+9·3495	5	47
2 12 17 40	8 46 18·92		118 56 39·5		−8·7207	+9·4065	8	45
8 11 29 2	8 28 16·81		119 53 15·4		−8·7335	+9·4063	4	48
8 11 29 2	8 28 16·69		119 53 16·4		−8·7335	+9·4063	4	49
12 10 41 51	8 15 43·95		120 17 24·1		−8·7560	+9·4480	10	50
12 10 41 51	8 15 44·09		120 17 22·6		−8·7560	+9·4480	10	51
13 10 42 47	8 12 32·35		120 21 49·2		−8·7480	+9·4254	10	51
13 10 42 47	8 12 32·34		120 21 50·8		−8·7480	+9·4254	10	52
14 10 6 56	8 9 24·37		120 25 1·1		−8·7750	+9·4961	8	51
14 10 6 56	8 9 24·51		120 25 2·8		−8·7750	+9·4961	8	52
15 10 51 49	8 6 4·97		120 28 14·0		−8·7201	+9·3604	5	50
15 10 51 49	8 6 4·99		120 28 10·2		−8·7201	+9·3604	5	51
15 10 51 49	8 6 4·45		120 28 10·1		−8·7201	+9·3604	5	53
18 10 0 25	7 56 32·41		120 32 3·8		−8·7558	+9·4398	5	54
18 10 0 25	∗ − 0 25·62		∗ + 14 13·6		−8·7558	+9·4398	5	55
19 12 29 27	7 52 58·90		120 31 53·1		−8·3980	+8·9230	6	54
19 12 29 27	∗ − 3 59·04		∗ + 14 3·1		−8·3980	+8·9230	6	55
28 9 56 41	7 25 5·76		119 58 23·8		−8·6586	+9·2644	6	56
28 9 56 41	7 25 6·21		119 58 27·2		−8·6586	+9·2644	6	57

Windsor Mean Time.	Comet's Apparent R.A.	$\Delta\alpha$	Comet's Apparent N.P.D.	$\Delta\pi$	Parallax Factors for R.A. Log, $\frac{p}{P}$	Parallax Factors for N.P.D. Log, $\frac{q}{P}$	No. of Comp.	Comp. Star.
d h m s	h m s		° ′ ″					
1882, Dec. 29 10 39 37	7 22 0·04		119 51 19·		−8·5397	+9·1086	6	57
1883, Jan. 1 9 4 21	7 13 24 64		119 26 57·3		−8·6986	+9·3504	4	58
1 9 4 21	7 13 24 33		119 26 57·0		−8·6986	+9·3504	4	59
7 10 35 4	6 56 53·57		118 22 6·2		−8·3072	+9 0396	6	60
9 15 24 31	6 51 19·92		117 54 28 6		+8·7231	+9·4395	7	61
9 15 24 31	6 51 20·03		117 54 27·1		+8·7231	+9·4395	7	62
12 9 54 22	6 44 45·74		117 17 8·8		−8·3498	+9·1195	8	63
12 9 54 22	6 44 45·55		117 17 5·9		−8·3498	+9·1195	8	64
14 9 52 53	6 40 15 66		116 48 39·3		−8·2901	+9·1271	4	65
Feb. 6 10 10 50	6 4 16·54		110 43 58·6		+8·2620	+9·3691	4	66
6 10 10 50	6 4 16·41		110 43 54·1		+8·2620	+9·3691	4	67
7 10 10 45	6 3 19 00		110 28 10·3		+8·2888	+9·3810	10	68
7 10 10 45	6 3 19·06		110 28 22·5		+8·2888	+9·3810	10	69
12 10 39 53	5 59 7·48		109 10 2·4		+8·4987	+9·4603	12	70
12 10 39 53	5 59 6·74		109 9 55·7		+8·4987	+9·4603	12	71
26 8 33 14	5 52 20·83		105 49 51·1		+8·2060	+9·4927	4	72
26 8 33 14	5 52 20·99		105 49 52·1		+8·2060	+9·4927	4	73
26 8 33 14	5 52 20·53		105 49 52·2		+8·2060	+9·4927	4	74
26 8 33 14	5 52 20·92		105 49 46·4		+8·2060	+9·4927	4	75
Mar. 2 9 1 48	5 51 29·44		104 57 23·8		+8·4237	+9·5330	5	76

Mean Places of the Comparison Stars for the Beginning of the Year of Observation, and the Reductions to the Apparent Places for the Days of Observation.

Star.	Mean R.A.	Reduction. +	Mean N.P.D.	Reduction. +	Authority.
	h m s	s	° ′ ″	″	
1	10 14 44·00	2·03	95 36 29·8	11·4	Lalande, 20075.
2	10 19 50·51	{2·03 / 2·05}	96 27 57·8	{11·5 / 11·5}	Radcliffe Obs. 1858, 688; Wash. Cat. 1860, 4336; Gr. 7 Yr. Cat. 1864, 1265; Bruxelles Cat. 1874, 762.
3	10 35 57	2·04	97 26 20	11·7	Equatorial Obs.
4	10 33 35	2·05	97 20 55	11·7	Equatorial Obs.
5	10 36 32·87	2·05	98 6 32·5	11·7	Lalande, 20677.
6	10 40 16·80	2·07	99 5 34·4	11·6	„ 20757.
7	10 43 48·94	2·07	99 13 32·8	11·7	„ 20850.
8	10 33 55	2·09	99 5 10	11·5	Star 9 mag., Equatorial Obs.
9	10 33 3	2·11	99 32 7	11·4	„ „ „
10	10 28 25·94	2·12	99 17 25·5	11·3	Wash. Cat. 1860, 4401.
11	10 30 25·02	2·13	99 58 13·2	11·3	Lalande, 20521.
12	10 31 20·99	2·18	101 8 5·1	11·2	„ 20545.
13	10 4 20·80	2·29	102 13 58 6	10·2	Radcliffe Obs. 1873, 506; Bruxelles Cat. 1873, 1596 and 1597; Cape Cat. 1880, 5507.
14	10 20 4·97	2·26	102 32 8·1	10·7	Lalande, 20242.
15	10 25 12·12	2·27	102 58 59·8	10·8	„ 20389.
16	10 5 49	2·36	103 42 12	10·1	Star 8½ mag., Equatorial Obs.

Star.	Mean R.A.	Reduction. +	Mean N.P.D.	Reduction. +	Authority.
	h m s	s	° ′ ″	″	
17	10 2 53·79	2·43	105 2 3·7	9·8	Arg. Oeltzen, 10397 and 10398.
18	10 3 27·09	2·43	104 54 28·0	9·9	,, 10403 and 10404.
19	10 15 8·79	2·40	104 54 19·3	10·3	Lalande, 20089.
20	10 10 44·24	2·48	106 11 16·2	10·0	Arg. Oeltzen, 10490.
21	10 20 23·01	2·45	106 14 4·8	10·3	Nautical Almanac, 1882.
20	10 10 44·24	2·50	106 11 16·2	10·1	Arg. Oeltzen, 10490.
21	10 20 23·01	2·47	106 14 4·8	10·4	Nautical Almanac, 1882.
22	9 58 25·19	2·61	107 31 49·7	9·5	Arg. Oeltzen, 10327.
23	9 55 46	2·72	109 6 44	9·3	Star 8 mag., Equatorial Obs.
24	10 1 46·62	2·70	109 8 1·3	9·5	Arg. Oeltzen, 10381.
25	10 2 46·78	2·69	109 9 53·1	9·6	,, 10396.
26	9 52 15·56	2·83	110 37 7·7	9·1	,, 10252.
27	9 43 20·91	2·95	111 28 15·7	8·9	Lalande, 19275.
28	9 49 37·39	2·92	111 46 41·3	9·1	Arg. Oeltzen, 10212.
29	9 49 3·68	2·92	111 55 50·9	9·0	,, 10206.
28	9 49 37·39	2·96	111 46 41·3	9·2	,, 10212.
29	9 49 3·68	2·95	111 55 50·9	9·1	,, 10206.
30	9 33 42·32	3·07	112 33 46·4	8·7	,, 9965 and 9966.
31	9 37 27	3·05	112 25 39	8·8	Star 7½ mag., Equatorial Obs.
32	9 45 36·86	3·02	112 27 56·7	9·1	Arg. Oeltzen, 10163.

Star.	Mean R.A. h m s	Reduction. + s	Mean N.P.D. ° ′ ″	Reduction. + ″	Authority.
33	9 35 53·99	3·09	113 3 19·3	8·7	Arg. Oeltzen, 10014, 10015 and 10016.
34	9 29 7·96	3·23	114 0 20·2	8·8	„ 9859.
35	9 31 14·89	3·22	114 10 36·9	8·8	„ 9911 and 9912; Wash. Cat. 1860, 4011; Cape Cat. 1880, 5183.
36	9 16 16·15	3·39	115 27 50·1	8·6	Arg. Oeltzen, 9623 and 9624; Gr. Obs. 1859, 263; Wash. Cat. 1860, 3923; Melb. Cat. 1870. 468; Cape Cat. 1880, 4996.
37	9 21 25·68	3·43	116 3 56·7	9·0	Arg. Oeltzen, 9725.
38	9 24 26·89	3·42	116 4 38·7	9·1	Wash. Cat. 1860, 3970; Wash. Obs. 1873, 214, and 1874, 64; Bruxelles Cat. 1873, 1497; Cape Cat. 1880, 5091.
39	9 24 40·27	3·42	116 4 24·4	9·1	Wash. Cat. 1860, 3974; Wash. Obs. 1873, 215, and 1874, 65; Radcliffe Obs. 1872, 465, and 1876, 368; Bruxelles Cat. 1873, 1498; Cape Cat. 1880, 5095.
40	9 23 26	3·43	115 49 54	9·1	Star 8 mag., Equatorial Obs.
41	9 10 49·26	3·52	116 42 17·1	8·8	Arg. Oeltzen, 9522; Wash. Cat. 1860, 3893; Cape Cat. 1880, 4935.
42	9 12 58·50	3·52	116 47 9·6	8·9	„ 9564.
43	8 54 15·21	3·82	118 20 55·3	9·5	„ 9215; Wash. Cat. 1860, 3798; Bruxelles Cat. 1873, 1425; Cape Cat. 1880, 4772.
44	9 0 4·84	3·80	118 32 52·5	9·7	Wash. Cat. 1860, 3832; Cape Cat. 1880, 4829.
45	8 45 5·45	3·85	119 1 28·6	9·2	Radcl. Obs. 1874, 507; Wash. Cat. 1860, 3726; Cape Cat. 1880, 4676.
46	8 45 25·64	3·85	118 49 49·8	9·2	Wash. Mural Cir. Zone, 102, 56.
47	8 51 20·52	3·83	118 59 58·9	9·4	Wash. Merid. Tr. Zone, 215, 48; Wash. Mur. Cir. Zone, 102, 59.
45	8 45 5·45	3·88	119 1 28·6	9·4	Radc. Obs. 1874, 507; Wash. Cat. 1860, 3726; Cape Cat. 1880, 4676.
48	8 24 28·68	4·09	119 58 24·6	10·0	Arg. Oeltzen, 8610; Wash. Cat. 1860, 3435.
49	8 25 2·51	4·09	119 55 52·0	10·0	„ 8620; Wash. Cat. 1860, 3440; Cape Cat. 1880, 4401.

Star.	R.A.	Reduction. +	Mean N.P.D.	Reduction. +	Authority.
	h m s	s	° ′ ″	″	
50	8 10 16·45	4·22	120 29 52·2	10·5	Arg. Oeltzen, 8280; Wash. Cat. 1860, 3332.
51	8 11 8·75	{4·21 / 4·24}	120 33 50·3	{10·5 / 10·8}	„ 8295; Wash. Cat. 1860, 3338; Cape Cat. 1880, 4236.
52	8 18 44·30	4·23	120 25 19·2	11·1	„ 8484; Wash. Cat. 1860, 3390.
51	8 11 8·75	4·27	120 33 50·3	11·1	„ 8295; Wash. Cat. 1860, 3338; Cape Cat. 1880, 4236.
52	8 18 44·30	4·26	120 25 19·2	11·4	„ 8484; Wash. Cat. 1860, 3390.
50	8 10 16·45	4·29	120 29 52·2	11·4	„ 8280; Wash. Cat. 1860, 3332.
51	8 11 8·75	4·29	120 33 50·3	11·4	„ 8295; Wash. Cat. 1860, 3338; Cape Cat. 1880, 4236.
53	8 15 20·26	4·28	120 43 35·3	11·5	Wash. Cat. 1860, 3364; Cape Cat. 1880, 4278.
54	7 50 34·28	4·38	120 36 28·9	11·5	„ 1880, 3240.
55	7 56 54	4·38	120 18	11·8	Star 8 mag., Equatorial Obs.
54	7 50 34·28	4·40	120 36 28·9	11·8	Wash. Cat. 1860, 3240.
55	7 56 54	4·40	120 18	12·1	Star 8 mag., Equatorial Obs.
56	7 19 4·55	4·56	120 13 17·2	13·2	Arg. Oeltzen, 6742; Radc. Obs. 1858, 529; Wash. Cat. 1860, 3006; Greenw. 7 Yr. Cat. 1864, 928; Bruxelles Cat. 1873, 1141; Cape Cat. 1880, 3620.
57	7 19 17·57	{4 56 / 4·57}	119 59 17·0	{13·2 / 13·6}	Arg. Oeltzen, 6750; Bruxelles Cat. 1874, 400; Wash. Obs. 1875, 322; Cape Cat. 1880, 3624.
58	7 12 23·97	2·26	119 31 16·9	7·9	Wash. Cat. 1860, 2945.
59	7 18 21·71	2·26	119 31 33·4	7·8	Arg. Oeltzen, 6721.
60	6 56 5·03	2·28	118 19 27·2	10·2	„ 6051 and 6052; Wash. Cat. 1860, 2834; Cape Cat. 1880, 3358.

Star.	Mean R.A.	Reduction. +	Mean N.P.D.	Reduction. +	Authority.
	h m s	s	° ′ ″	″	
61	6 54 55·65	2·29	117 43 50·1	10·8	Wash. Cat. 1860, 2827; Cape Cat. 1880, 3341.
62	6 57 3·52	2·29	117 46 4·9	10·8	Arg. Oeltzen, 6085 and 6086; Greenw. 6 Yr. Cat. 1850, 530; Radcliffe Obs. 1858, 499; Wash. Cat. 1860, 2845; Wash. Obs. 1875, 301; Cape Cat. 1880, 3370.
63	6 40 11·74	2·26	117 13 55·0	12·0	Arg. Oeltzen, 5615 and 5616; Wash. Cat. 1860, 2729; Wash. Obs. 1873, 161, and 1874, 36; Cape Cat. 1880, 3179.
64	6 43 54·35	2·27	117 15 2·4	11·9	Arg. Oeltzen, 5718, 5719, 5720 and 5721; Wash. Cat. 1860, 2747; Cape Cat. 1880, 3216.
65	6 48 18·32	2·28	116 48 41·2	12·4	Wash. Merid. Tr. Zone, 210,9; Wash. Mur. Cir. Zone, 152,8.
66	6 4 13·04	1·97	110 39 45·3	16·9	Arg. Oeltzen, 4697.
67	6 13 59·30	2·02	110 52 42·3	17·0	Lalande, 12118; Arg. Oeltzen, 4937 (Arg. Oeltzen R.A. rejected).
68	6 4 45·43	1·96	110 28 54·2	17·0	,, 11775; Arg. Oeltzen, 4709 and 4710.
69	6 10 6·41	1·99	110 14 20·1	17·0	,, 11984.
70	6 2 36·23	1·89	109 9 11·8	17·5	Arg. Oeltzen, 4665, 4667 and 4668; Greenw. 7 Yr. Cat. 1864, 787; Bruxelles Cat. 1873, 932; Cape Cat. 1880, 2801.
71	6 3 4·55	1·89	109 24 56·4	17·5	Lalande, 11721; Arg. Oeltzen, 4679 (1′ subtracted from Arg. Oeltzen N.P.D.).
72	5 49 7·99	1·61	105 45 20·9	17·8	Arg. Oeltzen, 4416.
73	5 50 53·24	1·62	105 32 28·5	17·8	,, 4448.
74	5 54 30·17	1·64	105 48 50·6	18·0	Lalande, 11409.
75	5 56 22·99	1·65	105 35 27·6	17·9	Arg. Oeltzen, 4541 and 4542.
76	6 0 51·60	1·61	104 55 32·9	18·2	,, 4621; Greenw. Obs. 1859, 169; Cape Cat. 880, 2780.

Windsor, N. S. Wales:
1883, *March* 6.

Note on Photographs of κ Crucis, η Argus, and the Nebula in Orion, and on Longitude Determinations at Melbourne. By R. L. J. Ellery, F.R.S.

(*Extract from a letter to Mr. Stone.*)

I forward herewith photographs, obtained with the Great Telescope, of *κ Crucis*, *η Argus*, and the nebula in *Orion*, for the Royal Astronomical Society. We have lately been experimenting with emulsion plates, and these are some of our first results, which I think are promising. You will notice in *κ Crucis* the great difference in size of the image in the three coloured stars which appear nearly of the same magnitude in the telescope.

Our longitude operations Singapore, Port Darwin, and Banjoewangie, as well as Port Darwin, Adelaide, and Melbourne, are satisfactorily concluded. The mean differences (which are, however, yet subject to small corrections for relay time, &c.) are wonderfully consistent and close. You will hear more of this next mail.

Observatory, Melbourne:
1883, *March* 15.

[The photographs have been placed in the Society's library.]